# Sequence and series

1, 3, 5, 7, -----
2, 4, 8, 16, 32, ----
1, 2, 3, 4, 5, ----
1/1, 1/2, 1/3, 1/4, ----

It is called sequence $\boxed{\text{or}}$ Progression (આવતી term)

1+3+5+7+----
2+4+8+16+----

It is called series. $\boxed{\text{or}}$ Sum of Progression (term ની add કરવામાં આવે)

## ☆ A.P. or Arithmatic Progression

Definition = An arithmatic progression (A.P.) is a sequence whose terms increases or decreases by a fixed number is called common difference.

$$a,\ a+d,\ a+2d,\ a+3d,\ ----,\ a+(n-1)d$$

$$n^{th} \text{ term of an A.P.} = \boxed{a+(n-1)d}$$

$$d = \text{common difference} = T_n - T_{n-1}$$

એક ટર્મ માંથી તેની આગળની ટર્મ બાદ કરવી

## Sum of first n terms

$a, a+d, a+2d, ----, \underbrace{a+(n-1)d}_{l = \text{last term}}$

$$S_n = \frac{n(a+l)}{2} \leftarrow n \text{ term} \times \text{તેની mean value}$$

$$S_n = \boxed{\frac{n}{2}\left[2a+(n-1)d\right]} \leftarrow l \text{ ની જગ્યા પર તેની term મુકતા}$$

$$\left(\frac{a+l}{2}\right) \leftarrow \text{mean of terms}$$

જો $S_n$ પરથી last term find કરવી હોય તો તેના માટે

$$\boxed{T_n = S_n - S_{n-1}} \rightarrow$$

આખી series ના sum માં last term નો sum નો હોય તો તે રીતે આખી series નો sum લઈ તેની આગળની સીરીઝ નો sum બંનેને બાદ કરતા, $T_n$ મળે.

**Ex** 1, 2, 3, 4, 5, 6, 7

અહી last term = 7

બધીનો સરવાળો = 28

તેની આગળની અંકસંખ્યા સુધીનો સરવાળો = 21

$$\left.\begin{aligned} T_n &= S_n - S_{n-1} \\ &= 28 - 21 = \boxed{7} \end{aligned}\right\}$$

આ રીતે last term ને સરવાળા પરથી મળી શકે.

## Arithmetic mean (A)

$$A = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n}$$

A.M. of two numbers $= \frac{a+b}{2} = A$

$\Rightarrow$ If $n$ arithmatic means between $a$ and $b$

મતલબ $a, A_1, A_2, A_3, A_4, \cdots, A_n, b$

This is a.P. of $(n+2)$ terms

આની common difference શોધવી

$T_n = a + (n-1)d$

$b = a + (n+1)d$

$d = \frac{b-a}{n+1}$

$T_n = b$

$n = n+2$

આનો ઉપયોગ કરતા

$A_1 = a + d$

$= a + \frac{b-a}{n+1} = \frac{na+b}{n+1}$

## Few notes

$a_1, a_2, a_3, a_4, \cdots, a_n$ are in A.P. then

① કોઈક વસ્તુ add અથવા subtract કરતા

$a_1 + \alpha, a_2 + \alpha, a_3 + \alpha, \cdots, a_n + \alpha$ is also A.P.

OR

$a_1 - \alpha, a_2 - \alpha, a_3 - \alpha, \cdots, a_n - \alpha$ is also A.P.

② કોઈક વસ્તુ ગુણતા અથવા ભાગતા

$ka_1, ka_2, ka_3, \cdots, ka_n$ is also A.P.

$\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, \frac{a_4}{k}, \cdots, \frac{a_n}{k}$ is also A.P.

(3) $a_1, a_2, a_3, \ldots, a_{n-2}, a_{n-1}, a_n$

$\rightarrow a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2}$

પહેલુ અને છેલ્લુ પદનો સરવાળો

= બીજુ અને છેલ્લેથી બીજા પદનો સરવાળો

= ત્રીજુ અને છેલ્લેથી ત્રીજા પદનો સરવાળો ------

Problem Solving TIP

જો question માં હોય કે a, b, c એ A.P. માં છે

તો $a - b = \cancel{b}\, c - b$

$\boxed{b = \frac{a+c}{2}}$

પહેલામાંથી બીજુ બાદ કરીએ તેને બરાબર જ ત્રીજા માંથી બીજુ બાદ કરીએ તે થાય

(4) જો ત્રણ term આપેલી A, B, C are in A.P

Solve કરવા } $A, A+d, A+2d$ લઈ શકાય

or

$A-d, A, A+d$ લઈ શકાય.

જો ચાર term આપેલી હો A, B, C, D are in A.P

Solve કરવા } $A-3d, A-d, A+d, A+3d$

⇒ Sum of terms of two A.P.'s is also A.P.

If $a_1, a_2, a_3, \ldots, a_n$ are in A.P.

$b_1, b_2, b_3, \ldots, b_n$ are in A.P.

then $a_1+b_1, a_2+b_2, a_3+b_3, \ldots, a_n+b_n$ are also in A.P.

# Geometric Progression (G.P.)

**Definition** = A geometric Progression (G.P.) is a sequence If, the ratio of any term and its Just before term is a constant number. This constant number is called common ratio.

$a, b, c, d, \ldots$ are in G.P. then

$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} \ldots = r =$ Common Ratio.

$\Rightarrow a, ar, ar^2, ar^3, ar^4, \ldots, \underbrace{ar^{n-1}}_{n^{th}\text{ term}}$ $\Big\}$ $r \in R$, $r \neq 0$

$$\boxed{T_n = ar^{n-1}}$$

## Sum of n terms of G.P

(1) $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

$r$ વડે ગુણતાં $\qquad S_n$ = Sum of $n$ term of G.P

(2) $rS_n = ra + ar^2 + ar^3 + \ldots + ar^n$

1 અને 2 ની બાદબાકી કરતાં

$rS_n - S_n = ar^n - a$ $\Big\}$ $\because ar, ar^2 \ldots ar^{n-1}$ સુધી cancel થઈ જશે.

$S_n(r-1) = a(r^n - 1)$

$\Rightarrow S_n = \frac{a(r^n - 1)}{(r-1)}$ $\quad r>1$ $\Big|$ $S_n = \frac{a(1-r^n)}{(1-r)}$ $\quad r<1$

$S_\infty$ = Sum of infinite no. of terms of G.P.

$S_\infty = \infty$ If $|r| \geq 1$ ; $S_\infty$ when $|r| \leq 1$

$$\boxed{S_\infty = \frac{a}{1-r}}$$

$r^n \to 0$ when $n \to \infty$

when $|r| < 1$ or
$-1 < r < 1$

## Geometric means (G)

Geometric mean for any $n$ Positive numbers $a_1, a_2, a_3, \ldots, a_n$

$G$ = geometric means = $(a_1 \times a_2 \times a_3 \times \ldots \times a_n)^{\frac{1}{n}}$

G.M. of two numbers a b

$$\boxed{G = \sqrt{ab}}$$

Geometrical mean of a sequence is its middle term.

$a, ar, ar^3, ar^4 \ldots ar^{n-1}$

For example

$a, ar, ar^2, ar^3, ar^4$ } બધાનો ગુણાકાર કરો 5 times Root કાઢવા જવાબ $ar^2$ આવે.

→ જો બેકી સંખ્યામાં term હોય તો બે middle term હોય અને તે બંનેનો ગુણાકાર કરી Root કાઢતા middle term મળે.

★ જો a અને b વચ્ચે n geometric means હોય તો

$a, G_1, G_2, G_3, \ldots, G_n, b$ will be a G.P. with total $n+2$ terms.

$b \rightarrow ar^{n+1} = T_{n+2}$

$$r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$a = G_0$

$G_1 = ar$

$G_2 = ar^2, \ldots$

## Problem Solving with G.P.

If given a, b, c are in G.P. then

$$\frac{b}{a} = \frac{c}{b} \Rightarrow \boxed{b^2 = ca}$$

Properties

If $a_1, a_2, a_3 \ldots$ are in G.P. then

(1) $1/a_1, 1/a_2, 1/a_3, \ldots$ are in G.P. with common Ratio $= 1/r$

(2) $ka_1, ka_2, ka_3 \ldots$ are in G.P. (કોઈ constant multiply કરીએ તો તે G.P. માં જ)

જો કોઈ add અથવા subtract કરીએ તો તે G.P. માં ન રહે.

(3) $a_1^2, a_2^2, a_3^2 \ldots$ are in G.P. with C. Ratio $= r^2$

If given $a, b, c, d, e$ are in G.P

take $a, ar, ar^2, ar^3, \ldots$ in place of $a, b, c, d, e$

આમ લઈને solve કરી શકાય

also $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

1 may be assumed as five consecutive terms of a G.P

## Harmonic Progression (H.P.)

If sequence $a_1, a_2, a_3, \ldots$ are such that

$\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots$ are in A.P. then they are in H.P.

$$T_n = \frac{1}{\frac{1}{a_1} + (n-1)\left(\frac{1}{a_2} - \frac{1}{a_1}\right)}$$

☆ No formula yet found for sum of a H.P.

Harmonic of $n$ positive numbers $a_1, a_2, a_3, \ldots a_n$ is

harmonic mean $$\frac{1}{H} = \frac{1}{n}\left(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}\right)$$

If $a, b, c$ are in H.P then

$b = \frac{2ac}{a+c}$ where $b$ is H.M. of $a$ and $c$

like $b^2 = ac$ where $b$ is G.M. of $a$ and $c$

and $b = \frac{a+c}{2}$ where $b$ is A.M. of $a$ and $c$

If $n$ harmonic mean $H_1, H_2, H_3, \ldots, H_n$ are inserted between $a$ and $b$ then,

$$\frac{1}{H_p} = \frac{1}{a} + p\,d \qquad \text{where } d = \frac{a-b}{(n+1)ab}$$

$a, H_1, H_2, \ldots, b$ ] will be a H.P. of $n+2$ terms.

☆☆ Relation among A.M. (A), G.M. (G) and H.M. (H) of two Real Positive numbers $a, b$.

→ If $a < b \longrightarrow$
$a < A < b$
$a < G < b$
$a < H < b$

$$A = \frac{a+b}{2}, \quad G = \sqrt{ab}, \quad H = \frac{2ab}{a+b}$$

$$AH = \left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right) = ab = G^2$$

$$\boxed{G^2 = A \times H} \quad \text{—— (1)}$$

$$A - G = \frac{a+b}{2} - \sqrt{ab}$$

$$A - G = \frac{(\sqrt{a} - \sqrt{b})^2}{2} \geq 0$$

$$\therefore \boxed{A.M. \geq G.M.} \quad \text{—— (2)}$$

$\boxed{A > G > H}$ by $AH = G^2 \Rightarrow \frac{A}{H} = \frac{G}{H} > 1 \Rightarrow$

Imp. Result

Sum of $n$ natural no. $\rightarrow \frac{n(n+1)}{2}$

Sum of $n^2$ natural no. $\longrightarrow \frac{n(n+1)(2n+1)}{6}$

Sum of $n^3$ natural no.

$$\Sigma n^3 = \left[\frac{n(n+1)}{2}\right]^2 = (\Sigma n)^2$$