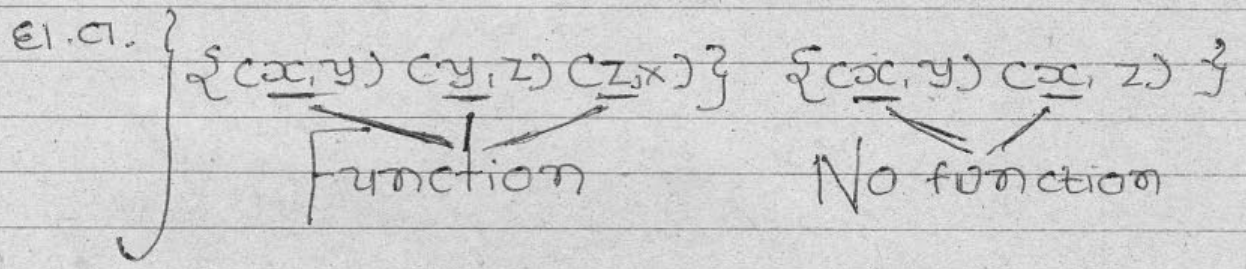




Function:-

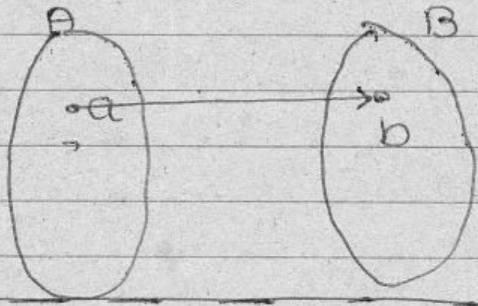
<Mapping>

In which no two order pairs have ~~same~~ same element

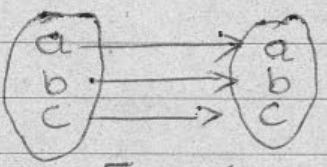


ਪੈਰਾ ਸੰਭਵਾ ਕਰਕਾ ਨ ਹੋਇ ਦੀ Function

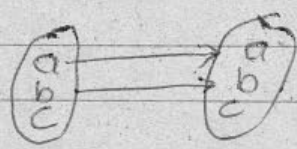
→ ਗਿਣਤੀ $f: A \rightarrow B$ (ਕਿਥਾ f is mapping from A to B)



→ Imp ਫੰਕਸ਼ਨ ਦੀ ਨਾਂ ਅਨੀ ਨੀ Set A ਦੀ ਅਥਾ ਕਿਉ ਨੀਕਾਰੀ ਹੋਇ



F V



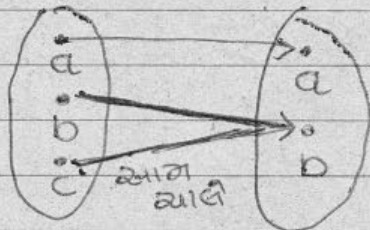
F X (ਕੋਈ)



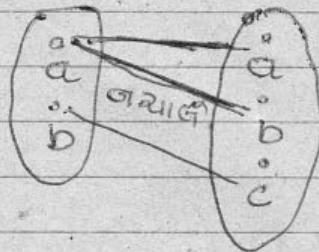
(2)

Page No. / /
Date: / /

→ ઘાન માં રાખવાના વૃક્ષ:



Function



No function

એક બિંદુ પહોલા set માં. એક વાર જ જોડાય.

Imp:-



(3)

Page No :

Date : / /

Relation

Relation मललल अक सैट अक सार्थी कौशल ली.....

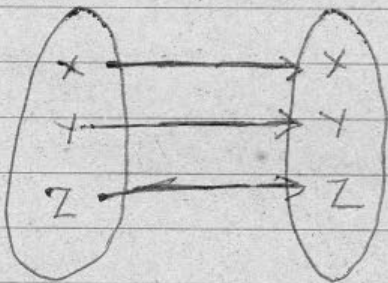
Difference between

Relation & Function

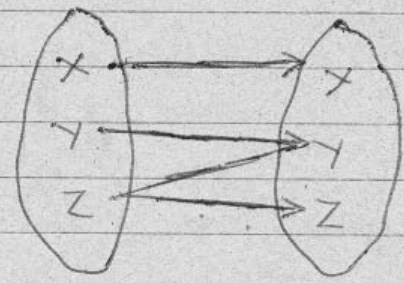
Imp.

संक्रान्त मां Set A मा अंकु अक क वार विलक्षण अनाली.

विलक्षण मां सार्थी लैट ल वार अनाली.



Function ✓
Relation ✓



Function ✗
Relation ✓

∴ Every Function Relation होय पल
" Relation Function न होय.

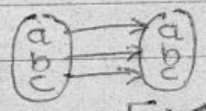


(4)

→ Function check કરવા તારીખા બધા
કુદા લાગુ પડવા કૌણબી

① Set A તા બધા બિંદુ જૌડાટીલ હૌય (B સાથે)

② Set A તા બિંદુ ઁક ક વાર જૌડાટીલ હૌય
ઁતીલના ંતે Image B તા હૌય

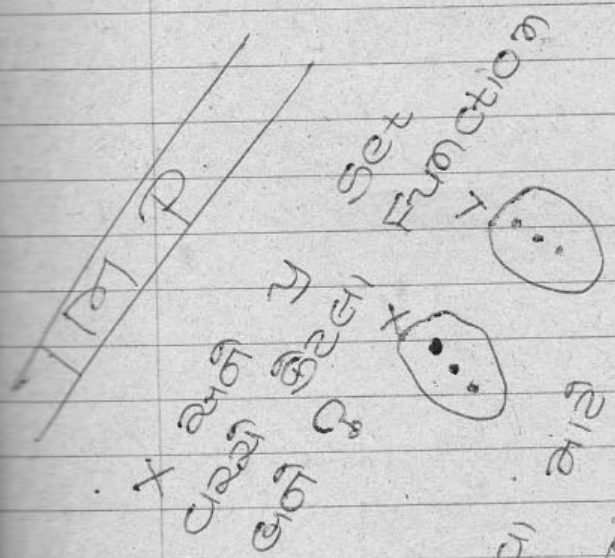


ઁથવા

③ Set A તા ઁથા વધારે બિંડ B તા ઁક
સત્ય સાથે જૌડાટીલ હૌય ઁલે



FV



X ઁકી ઁ
વઁબી કીડેલ
બીકે ઁ

ઁથવા કૌડ
 $n(X) = P$
 $n(Y) = Q$
then

~~P Q~~

કીડેલ
ઁકી
P Q
કીડેલ
Function બીકે
Relation

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→ Ordered Pair

(a, b) (b, c) are ordered pair केंद्रीय

$(a, b) \neq (b, a)$

(a, b)

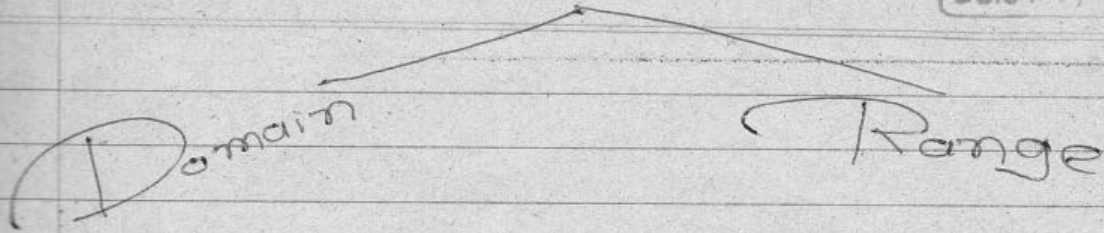
first Co-ordinate Second Co-ordinate

$(x, y) = (2, 3)$

then

$x = 2$

$y = 3$



every first Coordinate

every Second Co-ordinate

~~Ex~~

~~Relation~~

Ex: Relation $\{ (a, b) \underset{D}{\overset{R}{}}, (1, 2) \underset{D}{\overset{R}{}}, (3, 4) \underset{D}{\overset{R}{}}, (5, 6) \underset{D}{\overset{R}{}} \}$



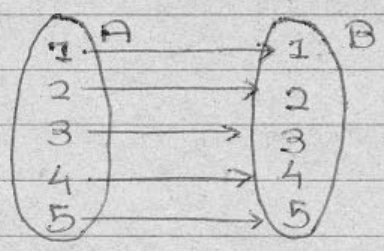
→ Co-domain:

Domain of Relation $\subseteq A$

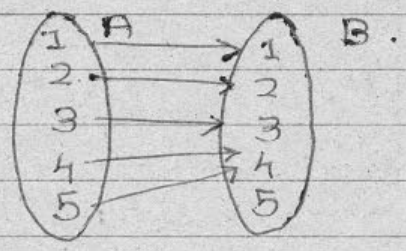
Range of Relation $\subseteq B$

उदा

जहाँ Range of Relation $\neq B$ then Biz Codomain



$R = B$



$R \neq B$ Biz



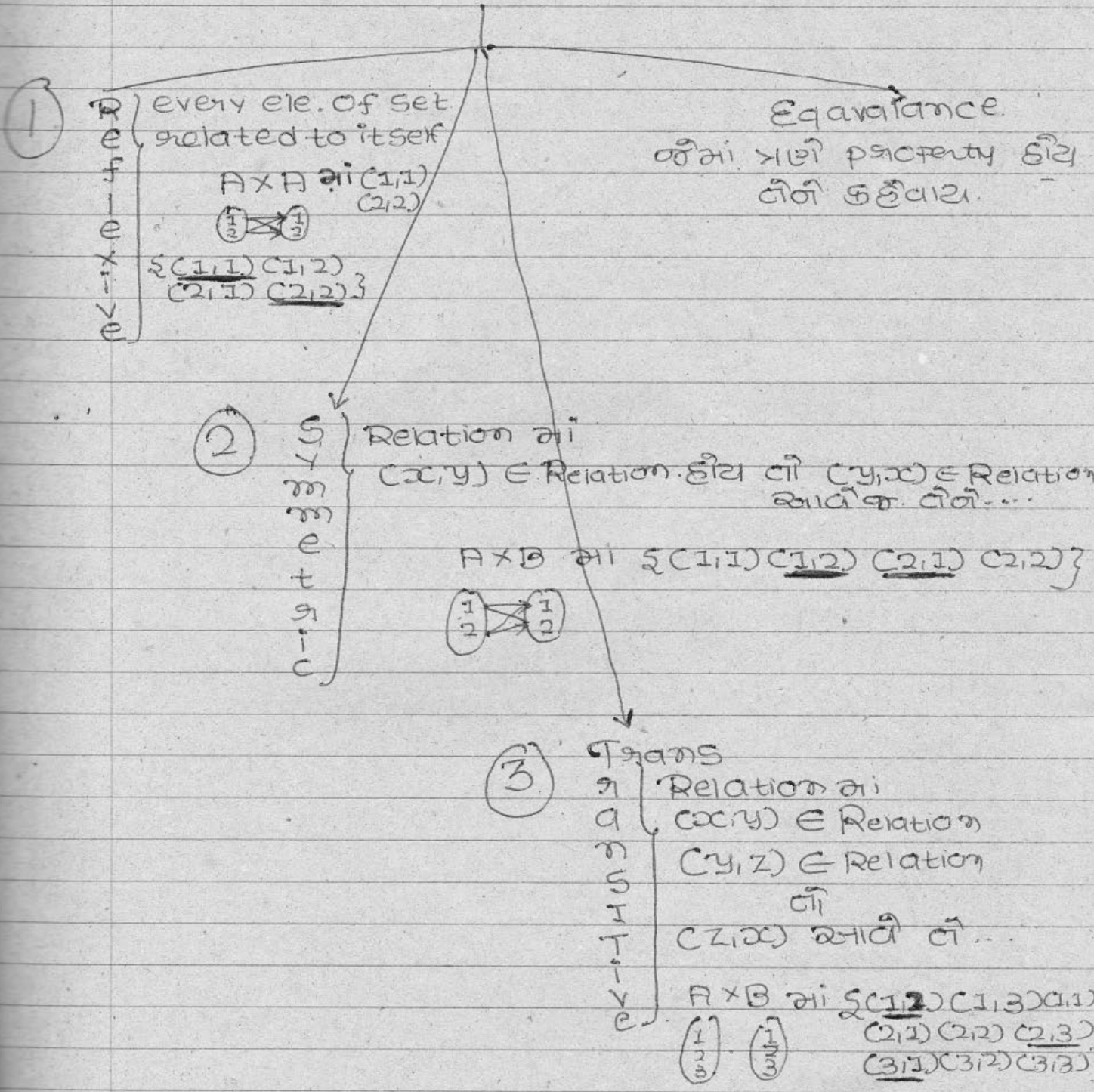
$D = \{1, 2, 3, 4, 5\}$
 $R = \{1, 2, 3, 4, 5\}$
 $=$
 $B = \{1, 2, 3, 4, 5\}$

$D = \{1, 2, 3, 4, 5\}$
 $R = \{1, 2, 3, 4\}$
 \neq
 $B = \{1, 2, 3, 4, 5\}$ Co-domain

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→ Property of Relation



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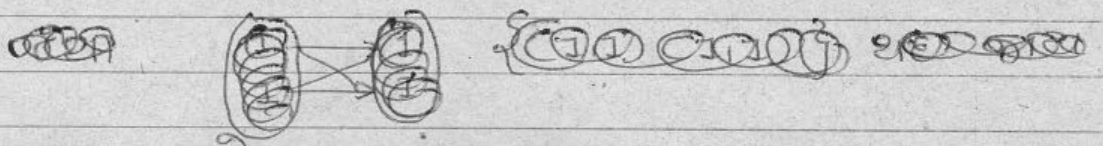
નોંધ:

→ આગળ આપેલ Symmetric Property

Anti Symmetric લાગે નહીં

જો $(x,y) \in \text{Relation}$ } માં
 $(y,x) \in \text{Relation}$

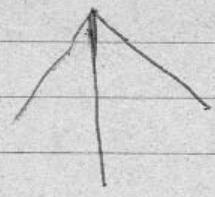
$x=y$ થાય/થઈ શકે



કારણ કે

જો $(x,x) \in \text{Relation}$ } અથવા
 $(x,x) \in \text{Relation}$

$(y,y) \in \text{Relation}$ થાય.
 $(y,y) \in \text{Relation}$



માત્ર આગળ આપેલ આ સંબંધ

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Type of Relation

① One to one

If Relation $\subseteq A \times B$
and
one element of A
only with
one element of B
then.

② one to many

If Relation $\subseteq A \times B$
and
one element of A
only with
more than one element of B

③ many to one

If Relation $\subseteq A \times B$
and
two or more element of A
with
only one element of B

④ many to many

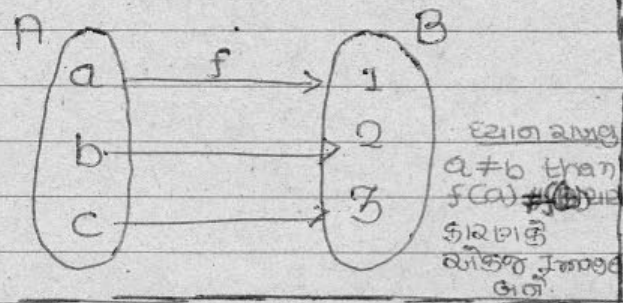
If Relation $\subseteq A \times B$
and
two or more of A
with
two or more of B



→ Type Of Function

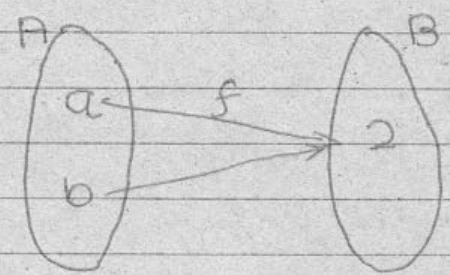
① One-One / Injective

मात्र एक-एक साथी च जोडाय



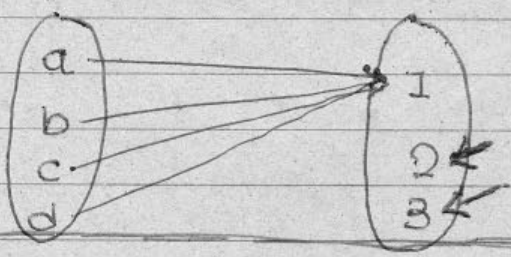
② Many-one

Two or more with one



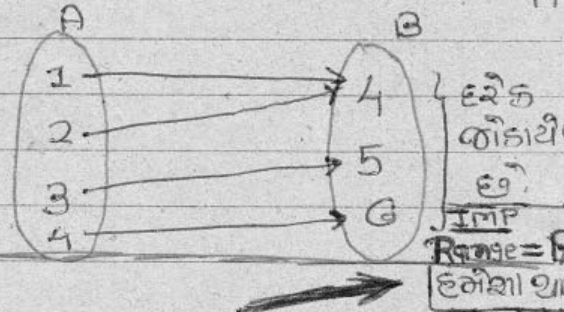
② Into

atleast one ele of B
no Image with ele of A



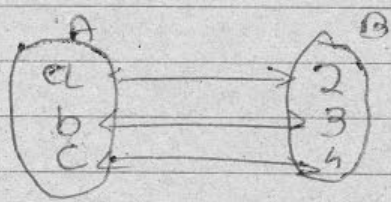
ONTO / Surjective

every ele of B
Image with atleast one ele of A



⑤ Objective function / bisection function. (bijective)

If function is Both one-one and onto





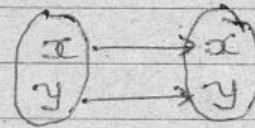
⑥ Identity Function / તાલેલ વિધેય :-

નિશાન I_A અથવા I

→ function define by $f(x) = x$

→ મતલબ કોઈકા element of A ના Image એ element પોતે જ હોય.

દા.ત. $f(x)$ ના x
 $f(y)$ ના y



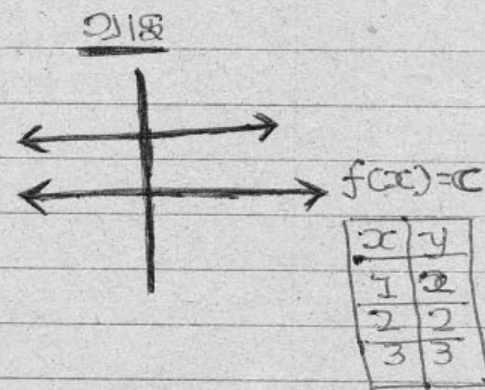
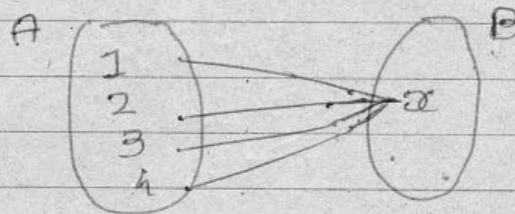
x	y
1	1
2	2
3	3

⑦ constant function / અચળ વિધેય :-

Range of function

Consist Only One element

દા.ત. $\{C_1(x), C_2(x), C_3(x), C_4(x)\}$



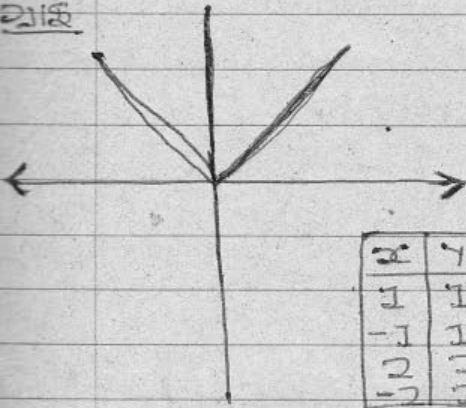
x	y
1	c
2	c
3	c



8) Modulus / भाणांक

फॉर्म: $f(x) = |x|$
 $x \in \mathbb{R}$

ग्राह



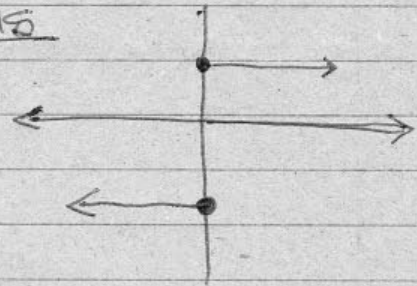
2	2
1	1
-1	1
-2	2

9) Signum / चिह्न

फॉर्म: $f(x) = 1$ if $x > 0$
 $= 0$ if $x = 0$
 $= -1$ if $x < 0$

Range always $\{-1, 0, 1\}$ था.

ग्राह



10) Greatest integer / अधतम पूर्णांक [Floor] 11) ceiling integer / अधतम पूर्णांक

फॉर्म: $f(x) = [x]$ ← closed interval

$[x] = -1$ for $-1 \leq x < 0$

$[x] = 0$ for $0 \leq x < 1$

$[x] = 1$ for $1 \leq x < 2$

$[x] = 2$ for $2 \leq x < 3$

फॉर्म: $f(x) = [x]$ ← close interval

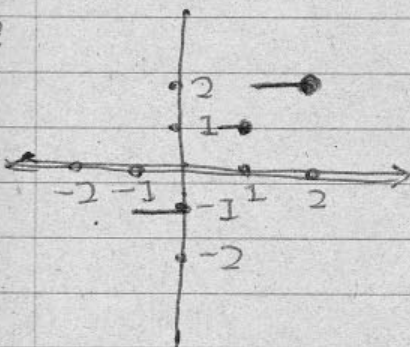
$[x] = 0$, $-1 < x \leq 0$

$[x] = 1$ $0 < x \leq 1$

$[x] = 2$ $1 < x \leq 2$

$[x] = 3$ $2 < x \leq 3$

ग्राह





(11) Real valued function :- ~~Real~~

→ Function which ^{define} a subset of Real Number set and its image also Real Number

उदा Physic ai speed of particle recordet az different time

$v = f(t)$ iz Real function.

x	x_1	x_2	x_3	x_4	...
$v = f(t)$	v_1	v_2	v_3	v_4	...

उदा

If domain A is subset of \mathbb{R}

or

If co-domain B is subset of \mathbb{R}

or

If A and B is subset of \mathbb{R}

A and B $\subseteq \mathbb{R}$

उदा

Domain and Co-Domain $\subseteq \mathbb{R}$

extra function

Page No. _____

Date: / /

12) Polynomial function (अष्टुपद)

Form:-

$$y = f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} \dots + a_{n-1}x + a_n$$

Where, $a_0, a_1, a_2, a_3 \dots a_n =$ real number $n =$ integer. (non-negative)

→ domain of Polynomial function is
always can be \mathbb{R} (Real Number)

Linear

Quadratic

Cubic

(i.e. Identity function)



$$y = ax^2 + bx + c \text{ या}$$

अर्थात्

$$y = ax^3 + bx^2 + cx + d \text{ या अर्थात्}$$

दो वास्तविक मूल

~~दो वास्तविक मूल~~

→ With two real root

~~two real root~~

→ With one real root

~~one real root~~

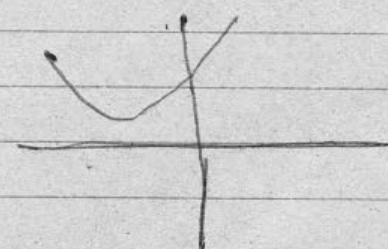
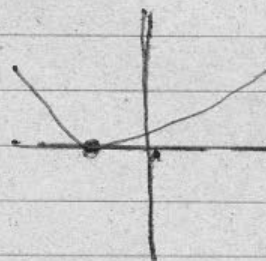
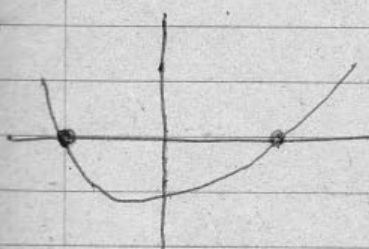
→ NO real root है।

~~NO real root है।~~

two real

one real

no real





Polynomial
function

✓

$$x^3 - x^2 + 2$$

✓

$$x^4 + \sqrt{2}x$$

✗

$$x^{\frac{2}{3}} + 2x \quad [\because n = \text{integer}]$$

< 11 English mein B. E. 1314 oibai >



(13) Rational function :- / रसिअ

Form $y = f(x) = \frac{P(x)}{Q(x)}$

$P(x), Q(x)$ = Polynomial function.

$f(x)$ = Rational function.

$$Q(x) \neq 0$$

< II English अरु (3-4) 15 अरु >



Function का प्रकार की ध्यान में रखकर
IMP कुछ :-

① Surjective function हीय ध्यारे हंमेशा।
Range = Set B थाय (∵ B में कुछ हीय है)।

② जो f onto/surjective न हीय ती हंमेशा।
~~into~~ into हीय [Function हीय ती एकका कही]
option [into, one-one] ?

③

④ Into function में Range ~~B~~ B हंमेशा थाय
C
(proper)
कारणकी B में हीक हीयपा पधारे जासकती

④



different मुल्ले:-

① Even function (बैकी संख्या)

जो x का कयाबी $-x$ मुकीबी ती पड।
function का हीर ती पड ती तीनी even कहैव।

दा.ग. $f(x) = x^2 + 2x^4$ $f(-x) = (-x)^2 + 2(-x)^4$
 $= x^2 + 2x^4$

$\cos(x)$

$\cos(-x)$

② Odd function :- (वैकी संख्या)

जो

जो x का कयाबी $-x$ मुकीबी ती हीर
पड ती तीनी odd कहैव।

दा.ग. $f(x) = x + 2x^3$ $f(-x) = (-x) + 2(-x)^3$
 $= -x - 2x^3$

$\sin(x)$

$\sin(-x) = -\sin(x)$



③ Equal function:

If domain & co-domain of

⊙

both function is equal

All domain & Codomain = Both Domain & Codomain.

④ Image :-

$$f : A \rightarrow B$$

$$a \in A$$

$$b \in B$$

Pre-Image
Inverse

Image

than

$$f(a) = b$$

⑤ Relation describe following five way

- ① Roster form or List form
- ② Set builder form
- ③ Arrow Diagram.
- ④ Tree diagram
- ⑤ Graphical Representation.



⑥ Inverse Relation

R = Relation then

R^{-1} = Inverse Relation

$$R^{-1} = \{(x, y) \mid x, y \in \text{Range}\}$$

$$R^{-1} = \{(y, x) \mid x, y \in \text{Range}\}$$

⑦ Inverse function

$f: A \rightarrow B$ then

$f^{-1}: B \rightarrow A$ = Inverse

B. E. = $f: A \rightarrow B$

$$f(x) = y = ax + b$$

$$f^{-1}(y) = x = \frac{y - b}{a}$$



Algebra of function

① Addition :-

$$\left. \begin{array}{l} f: x \rightarrow \mathbb{R} \\ g: x \rightarrow \mathbb{R} \end{array} \right\} (f+g): x \rightarrow \mathbb{R} \text{ and } x \in \mathbb{R}$$

$$(f+g)(x) = f(x) + g(x)$$

② Subtraction :-

$$\left. \begin{array}{l} f: x \rightarrow \mathbb{R} \\ g: x \rightarrow \mathbb{R} \end{array} \right\} (f-g): x \rightarrow \mathbb{R} \text{ and } x \in \mathbb{R}$$

$$(f-g)(x) = f(x) - g(x)$$

③ Multiplication

By scalar

$$f: x \rightarrow \mathbb{R}$$

$$\alpha = \text{scalar}$$

$$(\alpha f)(x) = \alpha f(x)$$

$$f: x \rightarrow \mathbb{R}$$

$$g: x \rightarrow \mathbb{R}$$

$$\left. \begin{array}{l} f: x \rightarrow \mathbb{R} \\ g: x \rightarrow \mathbb{R} \end{array} \right\} fg: x \rightarrow \mathbb{R}$$

$$(fg)(x) = f(x) \cdot g(x)$$



v> Quotient / division
 $x \rightarrow \mathbb{R} \quad \left\{ \begin{array}{l} x \in \mathbb{R} \end{array} \right.$

$$\left(\frac{f}{g} \right) (x) = \frac{f(x)}{g(x)} \quad \text{where } g(x) \neq 0$$

Pointz :- / NOTE :-

① Relation of A to B $\subseteq A \times B$

② If $f: A \rightarrow B$
 $g: B \rightarrow C$
 $h: C \rightarrow D$

then $g \circ f: A \rightarrow C$

प्रक्रिया $g \circ f(x) = g[f(x)]$

$f \circ g(x) = f[g(x)]$

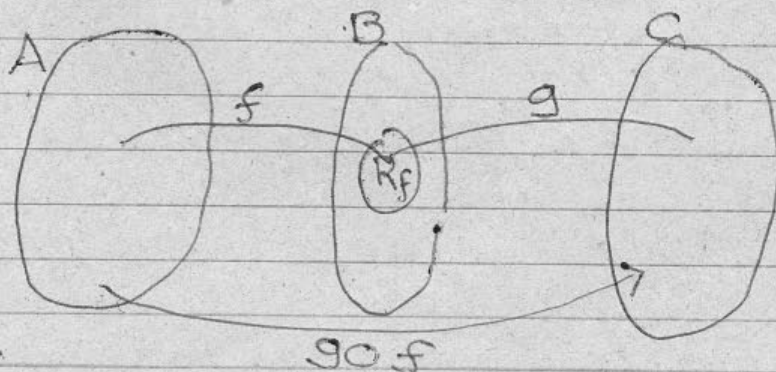
$h \circ (g \circ f)(x) = h[g(f(x))]$

$\rightarrow h \circ (g \circ f) = (h \circ g) \circ f$

$\rightarrow f \circ g = g \circ f$

उपर ना क्रिया ना विधीय क

संयोजन [composition of function] कहियेय.





→ આસ:-

905 બંને તે માટે $R_f \subset D_f$ હોય વ્યક્તિય

909 માટે $R_g \subset D_g$ હોય વ્યક્તિય

<વિષયનું સમજવા માટે ગુજરાતી P.9 બંને ઉદા. 1>

$f: A \rightarrow B$ તો $f \circ I_A = f$ અને

~~$I_B \circ f = f$~~ $I_B \circ f = f$ થાય

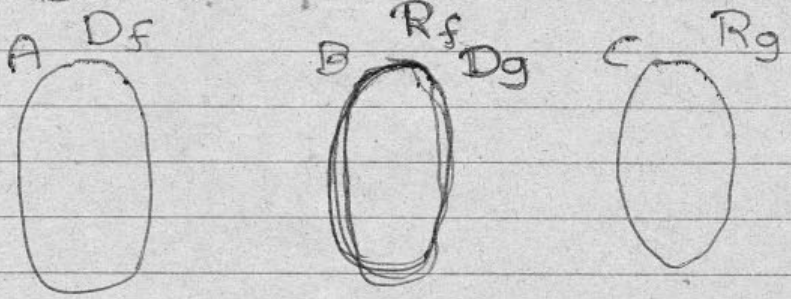
<આ સમજવા પ્રમેય 3.1 P.9 NO. 68 જુઓ>



મહત્વના મુદ્દા

$f: A \rightarrow B$ & $g: B \rightarrow C$ હોય ત્યારે

જો $g \circ f = R_f \cup D_g$ હોય તો કહવામાં
આવે છે કે f અને g એકબીજાની
સાથે સંબંધિત છે. આથી $g \circ f$
નું કોડોમનું સમૂહ A છે.



પણ $f \circ g$ માટે કોઈ પડે



Function

* Real No. \rightarrow 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, Decimal

EX 1.2,

3.0, 1.89564

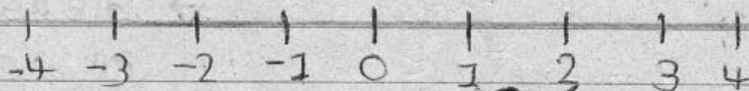
\rightarrow Real no. ની કીલ્ચા line પર બાંધી શકાય.

$\pi = 3.1415257 \dots$

$e = 2.71828 \dots$

} Irrational &

Real Numbers



[1 અને 2ની વચ્ચે infinite
no. હોય છે.
EX: 1.1, 1.001, 1.001...

* Complex No. \rightarrow Complex no. Real
અને Imaginary no.
બંને હોય.

$$i = \sqrt{-1}$$

Note

Real No.

Rational no. = $\frac{1}{2}, \frac{2}{3}, \frac{5}{2}, \frac{5}{4}, \dots$

જેની વચ્ચે Decimal Point
પણ fixed હોય

Irrational no. = 1.01..., 2.0012, ...

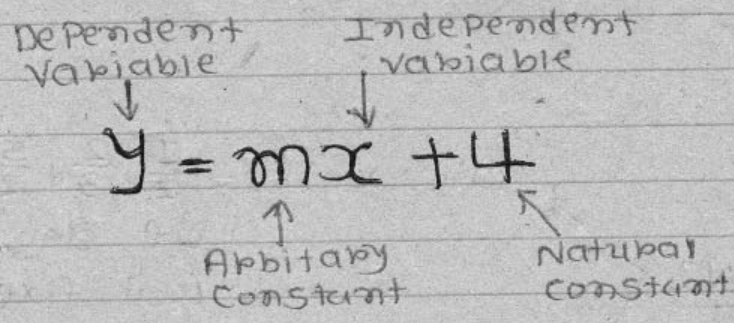
જેની વચ્ચે Decimal
Point પણ fixed
નહોતી

Variables

- Independent Variable \rightarrow આ Value કોઈના પર Dependent ની હીય.
- x
- Dependent Variable \rightarrow કોઈક ની ઉપર Dependent હીય, જો કોઈની Value બદલી તો આ Variable ની Value પણ બદલાય.
- y

Constants

- Natural constants \rightarrow કોઈયુદી equation માં Value fixed હીય.
- 1, 2, 3, π , e
- Decimal, $\overline{10}$ integer
- Arbitrary constants \rightarrow કોઈયુદી Particular equation માં Value fixed હીય, equation to equation બદલી શકે.
- a, b, c, d, A, B



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Function Definition.

$$f: x \rightarrow y$$

f maps x into y

Independent variable

Dependent variable

$$f: x \rightarrow y \quad : \quad \boxed{y = f(x)}$$

$$f: \mathbb{R} \rightarrow \mathbb{R}^+$$

Ex. $y = f(x) = x^2 + x + 1$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f: \mathbb{R} \rightarrow \mathbb{I}$$

$f(x, y) \rightarrow$ It is a function of x and y
Ex. $f(x, y) = x^2 + y^2 + xy$

NOTE

- $\rightarrow f(x)$ is
 - Real function \rightarrow Real no. સિવાય
 - Single variable \rightarrow એક જ variable
 - Single valued \rightarrow કોઈપણ એક જ વેલ્યુ

Ex. $y = f(x) = \sqrt{x}$
 $x = 4$
 $f(x) = \sqrt{4}$
 $= \boxed{2}$

એક function હોવાથી
 Single variable માટે
 Single value નો મતો
 તેથી $\sqrt{4}$ એ -2 અને $+2$
 બંને હોવા પછી તેને છોડી
 $+2$ ની બદલે $+2$ (\therefore Single value)

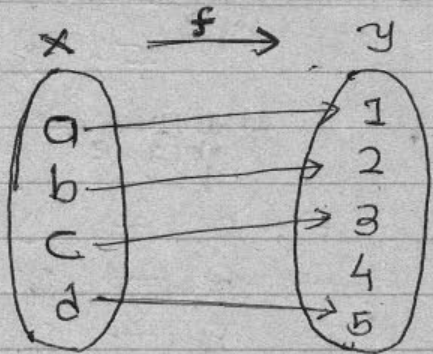


Domain, Co-Domain Range

→ The set of $x = \text{Domain}$
 $\{a, b, c, d\}$

→ The set of $y = \text{Co-domain}$
 $\{1, 2, 3, 4, 5\}$

→ The set of $f(x)$
 $\{1, 2, 3, 5\}$



∴ In General $f(x) \subseteq y$

Ex 3) $f: \mathbb{R} \rightarrow \mathbb{R} \quad y = \sqrt{x-2}$

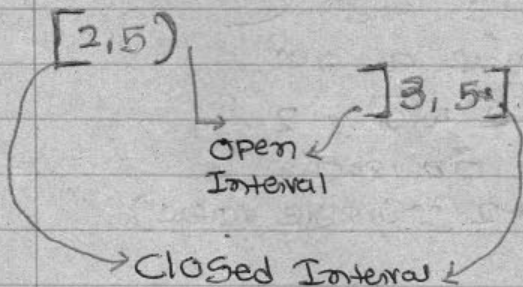
Domain = $x \geq 2$
 $= [2, \infty)$

Co-Domain = \mathbb{R}

Range = $y \geq 0$

$= [0, \infty)$

Ex 3) Real No. का एक Function
 $\therefore 2$ के बाद मान को $\sqrt{-}$
 धरने को Imaginary धरना



2 to 5 including 2

आइए

$\{2, 5\}$

की संख्या

2 0 5 के साथ

2 5 5 परमाणु आदि



Working Rule to find Domain of function, called D_f

① find Point where $f(x)$ becomes $\left(\frac{1}{0}\right)$ form.

② find Point where $f(x)$ becomes like $\sqrt{-ive}$

③ $\cos^{-1} x$, $\sin^{-1} x$ Here $-1 \leq x \leq 1$

④ $\operatorname{cosec}^{-1} x$, $\sec^{-1} x$ Here $(-\infty \leq x \leq -1) \cup (1 \leq x \leq \infty)$

③ and ④ Point on which value of x is not allowed

⑤ $\log x$ here $x > 0 \quad \therefore x \neq 0$

⑥ $\log_x f(x)$ $x > 0$ also $x \neq 1$
 $\frac{\log_e f(x)}{\log_e x}$



Solving with $\gg, \ll, >, <$ (Inequality) Date: / /

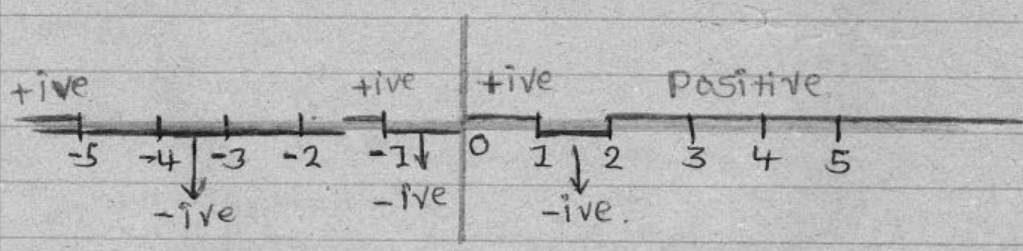
① $3 > 2$ so $a^3 > a^2$ if $a > 1$
EX $2^3 > 2^2 \rightarrow$ Base सरणी

$a^3 < a^2$ if $0 < a < 1$
EX $(\frac{1}{2})^3 < (\frac{1}{2})^2 \leftarrow$ Base सरणी.

② If $3 > 2$ $a^3 > a^2$ if a is +ive
 $3a < 2a$ if a is -ive

for $x > 0$ $3x > 2x$
 $3 > 2$

③ $\log_a 3 > \log_a 2$ if $a > 1$ } Base सरणी
 $\log_a 3 < \log_a 2$ if $a < 1$ }
here $a \neq 1$



$$G(x) = \frac{(x-1) x (x+1) (x-2)}{(x + \frac{3}{2}) (x+5)}$$



STEP-1 બધા Value જે Point પર 0 થતા હોય તે Point પર નીચાની કરવી.

STEP-2 આ Value ની Positive હોવાની જો સૌથી વધુ હોય તે સૌથી ઓછી માં પણ Negative હોવાની.

STEP-3 આ બધા Value માં Domain Positive માં Positive અને Negative માં Negative હોવાની.

Example find domain

① $f(x) = e^{1/x}$

$\frac{1}{x}$ is not defined for $x=0$ [since $\frac{1}{0}$ form $\frac{0}{0}$]
So Domain = $\mathbb{R} - \{0\}$

② $f(x) = \sqrt{(x-1)} + \sqrt{(6-x)}$

$$(x-1) \geq 0 \Rightarrow x \geq 1$$

$$6-x \geq 0$$

$$x \leq 6$$

$$\therefore D_f = [1, 6]$$