Quadratic Equation

$$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_n = 0$$

where, $a_0, a_1, a_2 \dots$ are constant

ની ~~...~~ Real અથવા imaganary કોઈ હોઈ શકે

આ પ્રકારના equation ને Polynomial Equation કહે છે

ની n degree વાળી છે

& Polynomial Equation of n degrees in x

Quadratic Equation માં કહેલા Real No. ના ઉકેલો આવે.

$n = 1 \rightarrow$ It is called Linear Equation ex. $ax + b = 0$

$n = 2 \rightarrow$ It is called quadratic equation ex. $ax^2 + bx + c = 0$

$n = 3 \rightarrow$ Cubical Equation. ex. $ax^3 + bx^2 + cx + d = 0$.

$n = 4 \rightarrow$ biquadratic equation. ex. $ax^4 + bx^3 + cx^2 + dx + e = 0$

આ રીતે Infinite ડિગ્રી equation બની શકે.

~~$(x-3)^2 = x^2 \dots = 0$~~

$(x-3)^2 = x^2 - 6x + 9 \} \leftarrow$ It is called identity

$\rightarrow$ It is true for all x.

$x^2 - 6x + 5 = 0 \} \leftarrow$ It is called equation

$\rightarrow$ It is true for only two values of x.

$f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_n = 0$

It is Polynomial

and $f(x) = 0$.

If $f(x)$ is of n degree then it will have n solutions

$f(x) = 0$ will be Possible for it's roots

if $x = a$ and $f(a) = 0$ $\} \rightarrow a$ is a root of Polynomial

★ n degree Polynomial will have exactly n roots or n solution.

$ax^2 + bx + c = 0$ and $a \neq 0$

It is quadratic and will have two roots:

let roots are α, β

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} ;$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha = \frac{-b + \sqrt{b^2-4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2-4ac}}{2a}$$

where b^2-4ac is called discriminant.
It is denoted by D or Δ
$\therefore D = b^2-4ac$

Note: અહીં b^2-4ac ને discriminant છે. $\sqrt{b^2-4ac}$ નહીં.

ઉપરના Equation નાં બીજો પ્રમાણે પણ લખી શકાય

$$\alpha = \frac{-b + \sqrt{D}}{2a}$$

$$\beta = \frac{-b - \sqrt{D}}{2a}$$

D ની જગ્યાએ Δ પણ લખી શકાય.

Relation between roots and coefficients

$$(x-\alpha)(x-\beta) = 0$$

Solve કરતા

$$x^2 - x(\alpha+\beta) + \alpha\beta = 0$$

$\downarrow$

$$\alpha+\beta = \frac{-b}{a}, \quad \alpha \times \beta = \frac{c}{a}$$

a વડે ભાગતાં મળતાં $(\alpha+\beta)$ root નાં coefficient અને $\alpha \times \beta$ root ના coefficient આવે.

---

for discriminant

1) If $b^2-4ac > 0$ then roots are unequal and real.

(i) If $\sqrt{b^2-4ac}$ is rational then roots are real, unequal and rational.

(ii) If $\sqrt{b^2-4ac}$ is irrational then roots are real, unequal and irrational.

2) If $b^2-4ac=0$ then roots are equal and

3) If $b^2-4ac<0$ then roots are imaginary (com

for Relation of Roots and coefficient

$$ax+by+c=0$$

તેનો slope find કરવા

$$\text{slope }(m) = -\frac{\text{coefficient of } x}{\text{coefficient of } y} = -\frac{a}{b}$$

Nature of Roots

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ If a, b, c are real

Case-1 $b^2 = 4ac$ means $D = 0$ $\left(\begin{array}{l} D = b^2 - 4ac \\ 0 = b^2 - 4ac \\ b^2 = 4ac \end{array} \right)$

$$\alpha, \beta = \frac{-b}{2a}$$ $\boxed{\text{Roots are Real and Equal}}$

Case-2 $b^2 < 4ac$ means $D = -ive$ (જો $b^2 < 4ac$ હોય ત્યારે જ કારણ કે Discriminant negative થાય)

$$\alpha, \beta = \frac{-b \pm iD}{2a}$$ આ રીતે $\sqrt{\ }$ ની અંદર ડિસ્ક્રિમિનન્ટ – હોય તેના લીધે i આવશે. એની અંદર ની D ના રીતે ધ્યાનમાં રાખી ની iD

Roots are complex number (∴ + હોય) અને conjugate (∴ – હોય) ની Pair કરી.

It means If $A + Bi$ is a root then other root will be $A - Bi$

Case-3 If $b^2 > 4ac \rightarrow D = +ive$

(i) $b^2 - 4ac$ is Perfect square

α, β will be real, distinct (unequal) and rational roots.

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

[Case-3 ની અંદર G ... Point આવે કે value Rational કે irrational એ નક્કી થાય.]

(ii) $b^2 - 4ac$ is not a Perfect square, α, β will be real, distinct (unequal) but irrational roots. will be of Pair

like $3 + \sqrt{5}$ and $3 - \sqrt{5}$ } (∴ ± ઉલટે નીચે પ્રમાણે જોડ બને)

આ Pair ગણવા પડશે $\rightarrow \frac{1}{5 + \sqrt{7}}$ and $\frac{1}{5 - \sqrt{7}}$ } $\rightarrow$ આવી ગણી Pair હોય છે અને ∴ irrational છે.

$\alpha, \beta = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$; $\alpha+\beta = \frac{-b}{a}$; $\alpha \times \beta = \frac{c}{a}$:

(i) If one root is zero then $\frac{c}{a} = 0 \Rightarrow c = 0$.

(ii) If root are of opposite sign $\frac{c}{a} < 0$

It means c and a are of opposite sign.

(iii) If both roots are zero $\frac{b}{a} = 0 \Rightarrow b = 0$ & $c = 0$

(iv) Roots will have same sign → c and a will have same sign (Positive)

($\because$ $- - \to +$ ve $+ + \to +$ ve)

(v) Roots are reciprocal $\alpha\beta = 1$ then $c = a$. $\left(\because \alpha\beta = \frac{c}{a},\ 1 = \frac{c}{a},\ \boxed{a = c}\right)$

(vi) Sum of root is 0 then $b = 0$. $\left(\because \alpha+\beta = 0,\ \therefore 0 = \frac{-b}{a} \;\middle|\; -b = 0,\ \boxed{b = 0}\right)$

*) If $a = 1$. $b, c \in I$ and roots of $ax^2+bx+c=0$ are ration no. then these root must be integers

*) If $ax^2+bx+c=0$ satisfied by more unequal complex number then $ax^2+bx+c=0$ becomes identity. $a=b=c=0$

$$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n = 0$$

Sum of Roots $\rightarrow$

$$\left.\alpha_1 + \alpha_2 + \dots + \alpha_n = (-1)^1 \frac{a_1}{a_0}\right\} \text{coefficient}$$

Sum of Product of two roots $\rightarrow$

$$\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_1\alpha_4 + \dots = (-1)^2 \frac{a_2}{a_0}$$

Sum of Product of three roots $\rightarrow$

$$\alpha_1\alpha_2\alpha_3 + \alpha_2\alpha_3\alpha_4 + \alpha_3\alpha_4\alpha_5 + \dots = (-1)^3 \frac{a_3}{a_0}$$

Product of all roots $\rightarrow$

$$\alpha_1\alpha_2\alpha_3 \dots \alpha_n = (-1)^n \frac{a_n}{a_0}$$

Note. $\alpha + \beta = -\frac{b}{a} \longrightarrow \alpha + \beta + \gamma$ is also $= -\frac{b}{a}$

$$\alpha\beta = \frac{c}{a} \longrightarrow \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

Symmetric functions of Roots

→ કોઈપણ Equationના બે Root α અને β હોય. તો α ની જગ્યાએ β અને β ની જગ્યાએ α મૂકીએ તો પણ equation માં કંઈ change ના થાય.

let $\alpha, \beta \rightarrow ax^2 + bx + c = 0$

$$\alpha + \beta = \frac{-b}{a} \quad , \quad \alpha\beta = \frac{c}{a}$$

(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$$= \frac{b^2}{a^2} - \frac{2c}{a}$$

$$= \frac{b^2 - 2ca}{a^2}$$

$\alpha+\beta$ અને $\alpha\beta$ ની value ઉપર આધારિત હોય છે

(ii) $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$$= \left(-\frac{b^3}{c^3}\right) - 3\times\frac{c}{a}\left(\frac{-b}{a}\right)$$

(iii) $\alpha^4 + \beta^4 = \left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 2\alpha^2\beta^2$

$$= \left[\left(\frac{-b}{a}\right)^2 - 2\times\frac{c}{a}\right]^2 - 2\left(\frac{c}{a}\right)^2$$

ની modulus કાઢ નો

$$|\alpha-\beta| = \sqrt{(\alpha+\beta)^2-4\alpha\beta}$$

$$= \sqrt{(\alpha-\beta)^2}$$

$$|\alpha^2-\beta^2| = (\alpha+\beta)\sqrt{(\alpha+\beta)^2-4\alpha\beta}$$

$$\boxed{\alpha^2\beta+\beta^2\alpha = \alpha\beta(\alpha+\beta)}$$

Common Roots

$$a_1x^2+b_1x+c_1=0$$
&
$$a_2x^2+b_2x+c_2=0$$

બંને સમીકરણોને satisfied કરે

One root common case

let α is common root.

then.

$$\therefore a_1\alpha^2+b_1\alpha+c_1=0$$

and

$$a_2\alpha^2+b_2\alpha+c_2=0$$

Cross multiplication કરતાં

$$\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{c_1a_2 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\alpha = \frac{c_1a_2 - a_1c_2}{a_1b_2 - a_2b_1} \; ; \; \alpha = \frac{b_1c_2 - b_2c_1}{c_1a_2 - a_1c_2}$$

let

$(x-\alpha)(x-\beta)=0$ } બંનેમાં

$(x-\alpha)(x-\gamma)=0$ } કોઈ Root α common

$x^2 - \alpha x - \beta x + \alpha\beta = 0$ } બંનેમાં x^2 α

$\alpha x^2 - \alpha x - \gamma x + \alpha\gamma = 0$ } રાખતાં પછી substitute કરતાં

$(x-\alpha)\,[\gamma - \beta) = 0$ | $x(\gamma-\beta)+\alpha(\beta-\gamma)=0$

$\boxed{\alpha = x}$ | $\boxed{x = \alpha}$

બંનેનું subtract કરતાં
Root નો common કોઈ નીકળી જાય

here α

If two roots are common

$$\alpha+\beta = \frac{-b_1}{a_1} = \frac{-b_2}{a_2} \; ; \; \alpha\beta = \frac{c_1}{a_1} = \frac{c_2}{a_2}$$

$\therefore$ $$\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

This is the condition for both root to be common

→ α+β અને αβ બંનેની કિંમતો Ratio માં હોય.

→ Main condition

બંને Equation ના બધા coefficient Ratio માં હોય. કિંમતોના

$\therefore$ minimum and maximum value of ax^2+bx+c

Case 1 $a>0$ ax^2+bx+c

$$= a\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a}$$

$\therefore$ minimum value $= \dfrac{4ac-b^2}{4a}$.

Case 2 $a < 0$

$$ax^2+bx+c = a\left(x+\frac{b}{2a}\right)^2 + \left(\frac{4ac-b^2}{4a}\right)$$

maximum value $= \dfrac{4ac-b^2}{4a}$

Imp both min and max occurs at

$$x+\frac{b}{2a}=0$$

જેના max x, જ્યાં min value ની $x+\frac{b}{2a}$ પરથી મળતી હશે

$$x = \frac{-b}{2a}$$

$\therefore$ $\left(\frac{\alpha+\beta}{2}\right.$ A. M. ના ...

$$\frac{\alpha+\beta}{2} \longrightarrow \text{A.M.} \left(\text{Arithmatic mean}\right)$$

Lagrange's Identity.

$$(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2) - (a_1b_1+a_2b_2+a_3b_3)^2$$

$$= (a_1b_2-a_2b_1)^2 + (a_2b_3-a_3b_2)^2 + (a_3b_1-a_1b_3)^2$$

$$\rightarrow \frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d} = \frac{a+c}{b+d} = \frac{(a^2+c^2)^{1/2}}{(b^2+d^2)^{1/2}}$$

Identity ના ઉપયોગ Polynomial Equation માં થાય

General Polynomial & relation of roots.

$(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)=0$ બંને equivalent

$ax^3+bx^2+cx+d=0$

આ બંને expand કરતા

$x^3+x(\alpha_1+\alpha_2+\alpha_3)+x(\alpha_2\alpha_3+\alpha_3\alpha_1+\alpha_1\alpha_2)$

$-\alpha_1\alpha_2\alpha_3=0$

Q.

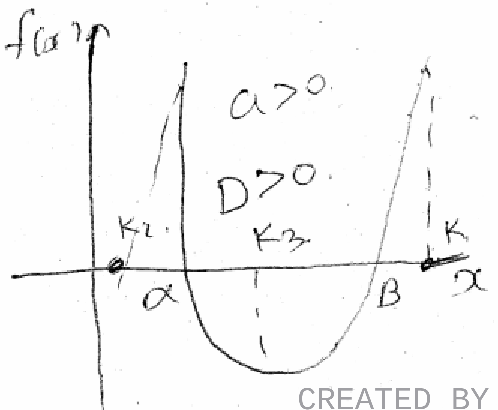
Imp ∴ $\alpha_1+\alpha_2+\alpha_3=\dfrac{-b}{a}$ ગમે તે order માટે લાગુ

$\alpha_2\alpha_3+\alpha_3\alpha_1+\alpha_1\alpha_2=\dfrac{c}{a}$ Equation માટે

$\alpha_1\alpha_2\alpha_3=\dfrac{-d}{a}$

Graphs

$f(x)=ax^2+bx+c$ $=0$ થયા ∵ function છે



D>0 હોય તો કારણ કે Root positive છે.

3,11 Parabola કરો. ... [illegible]

f(α) માં x ચાલે 0 છે.

$$\therefore \left.\begin{array}{l} f(\alpha) = 0 \\ f(\beta) = 0 \end{array}\right\} \alpha, \beta \text{ એ roots}$$

Graph જ્યાં x ને છેદ કરે ત્યાં જ.
ઉકેલો f(x) ના Root થાય.

→ K એવો number છે જ્યાં α અને β
નાના હોય કિંમત આવેલ છે.

$\therefore f(K) > 0$

also for K_2

$$\left.\begin{array}{l} \alpha, \beta > K_2 \\ \therefore f(K_2) > 0 \end{array}\right\}$$ કારણ કે positive હોવાથી x ની ઉપર છે.

$$\left.\begin{array}{l} \alpha < K_3 < \beta \\ \cancel{f(K_3) > 0}\ f(K_3) < 0 \end{array}\right\}$$ ∴ બંને Root ની અંદર છે

$$\therefore f\left(\frac{\alpha+\beta}{2}\right) = \text{minimum value}$$

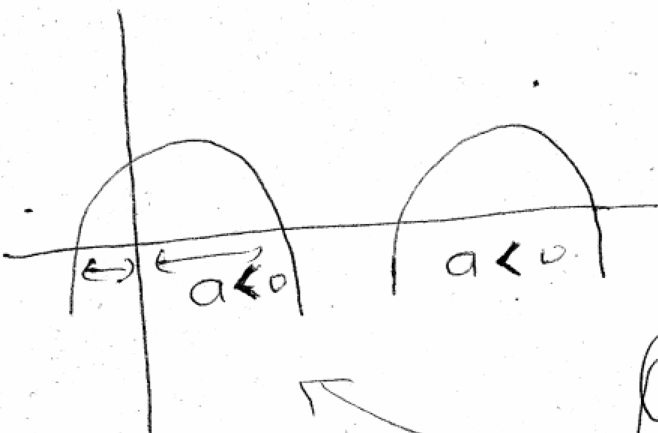
a ની value negative તો graph ઉંધો
a ની value positive તો graph સીધો

$f(0) > 0$ } આ condition હોય

$\therefore$ α, β એ x ની Positive side પર છે.

$\therefore$ ~~f>0~~

$\alpha, \beta > 0$



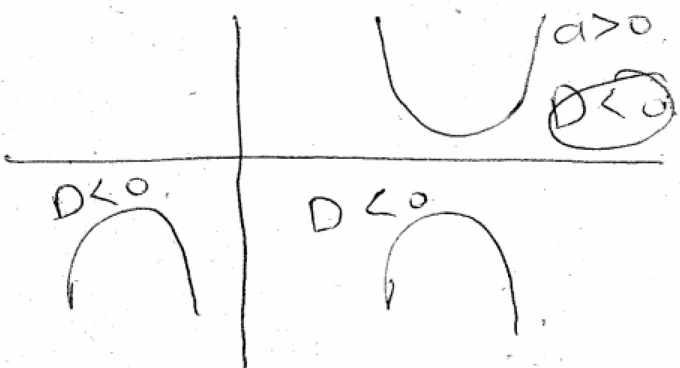
← here

$\alpha + \beta = 0$

$\rightarrow -\dfrac{b}{a}$

$b > 0$

$\because$ એક Root negative નો છે. એક Root positive નો છે.
a Positive છે b negative
$\therefore$ − − + થશે જેના
b ના Positive જ
રહેશે.



→ આ ડી ગ્રાફ ઉપયોગ જ્યારે
D ને 0 કરતાં નાનો
કરી.

$D = b^2 - 4ac$

Note

Root શોધવા કંઈક
ગ્રાફ f(x) ની cut
કરતો ડીગ્રી શોધીશું
જેના કરી તો Root
મળશે.

પરંતુ આ ડી બંને Root
શોધી શકાય Root
નથી મળતા.
ગ્રાફ x અક્ષને cut કરે
તો Root મળે પરંતુ
cut કરતો નથી. તો Root

 નથી.