Circle

General form of Equation

General 2 Degree equation } $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

આ પરથી circle નું equation બનાવવા cond. ① $a = b = 1$ ② $h = 0$, $\Delta \neq 0$

Circle Equation → $\boxed{x^2 + y^2 + 2gx + 2fy + c = 0}$

Note → જો x અને y માં કોઈ બીજી સંખ્યા તો આખા equation ને તે સંખ્યા થી Divide કરવું

ટૂંકમાં x અને y ના coefficient 1 રાખવા

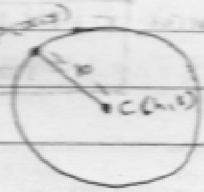
Imp જો x અને y ના coefficient માં કંઈક સંખ્યા હોય અને બંનેમાં તે સંખ્યા અલગ અલગ હોય તો તે circle represent નથી કરતું

Proof →

Circle નું centre → $(-g, -f)$

Radius → $\sqrt{g^2 + f^2 - c}$

Central form



main → $(x-h)^2 + (y-k)^2 = r^2$

→ $x^2 + h^2 - 2hx + y^2 + k^2 - 2yk - r^2 = 0$

→ $x^2 + y^2 + \underbrace{x(-2h)}_{2g} + \underbrace{y(-2k)}_{2f} + \underbrace{h^2 + k^2 - r^2}_{c} = 0$

$h = -g$

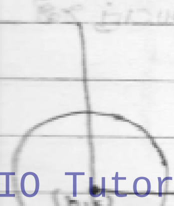
$k = -f$

$c = h^2 + k^2 - r^2$

$r^2 = h^2 + k^2 - c$

$r = \sqrt{h^2 + k^2 - c}$

Circle at origin



$h = 0$, $k = 0$

$(x-0)^2 + (y-0)^2 = r^2$

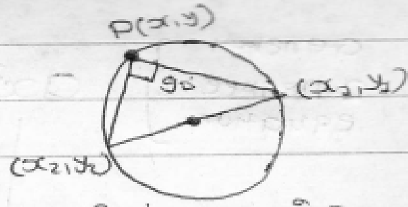
$x^2 + y^2 = r^2$ → Equation of circle at centre

Diameter form

$$\left(\frac{y_2-y}{x_2-x}\right)\left(\frac{y_1-y}{x_1-x}\right) = -1$$

(From $m_1 \cdot m_2 = -1$)

↑ slope perpendicular માટેની condition



જો Diameter નાં P પર
જોડવામાં આવે છે. આ
P point આગળ circle
માં ગમે ત્યાં 90° થાય.

main → $(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

Radius માટે $\Rightarrow r = \sqrt{g^2+f^2-c}$

① જો $g^2+f^2-c > 0$ તો $r = +$ive મતલબ $\boxed{\text{circle} = \text{Real}}$

② જો $g^2+f^2 = c$ તો $r = 0$ મતલબ $\boxed{\text{circle} = \text{Point circle}}$

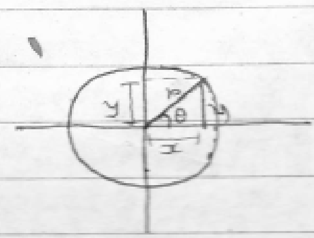
③ જો $g^2+f^2-c < 0$ તો $r = -$ive મતલબ $\boxed{\text{circle} = \text{imaginary}}$

Parametric form

જો circle origin પર છે.
તો h અને k = 0 થાય.

$x = r\cos\theta$

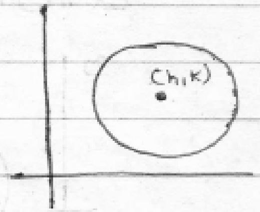
$y = r\sin\theta$



જો r = cons. છે. કા.કે. radius circle માટે હંમેશા cons.

θ = Parameter કા.કે. જેમ θ માં પરિવર્તન આવતું જશે
તેમ circle બનતું જશે.

જો circle origin પર નથી તો.
h અને k $\neq 0$ થશે.
તેથી તે માટે

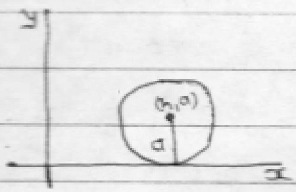


$$x = h + r\cos\theta$$
$$y = k + r\sin\theta$$

અહીં h અને k નું Distance અલગ અલગ હોય શકે પરંતુ r = cons. જ રહે.

Circle touching the axis

① કોઈપણ એક અક્ષને touch કરે



x અક્ષને touch કરે તો y = circle ની Radius થાય

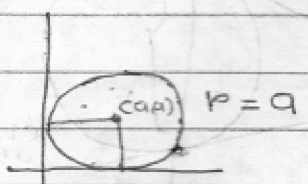
$$(x-h)^2 + (y-a)^2 = a^2$$
$$x^2 + h^2 - 2xh + y^2 + a^2 - 2ya = a^2$$
$$x^2 + y^2 + h^2 - 2xh - 2ya = 0$$

આવી રીતે જો y અક્ષને touch કરે

$h = a$, રહે. થાય. $k = k$ થાય

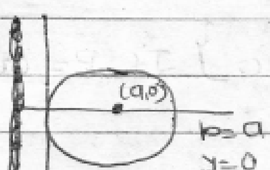
$$\therefore (x-a)^2 + (y-k)^2 = a^2$$

② Circle બંને axis ને touch કરે



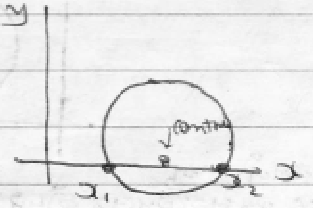
h = k = a

$$(x-a)^2 + (y-a)^2 = a^2$$
$$x^2 + a^2 - 2ax + y^2 + a^2 - 2ay = a^2$$
$$x^2 + y^2 - 2ax - 2ay + a^2 = 0$$



$$(x-a)^2 + (y-0)^2 = a^2$$
$$x^2 + y^2 - 2ax = 0$$

Length intercept made by the circle at axis



$$x^2+y^2+2gx+2fy+c=0$$

$$y=0$$

$$x^2+2gx+c=0$$

$$x_1 = \frac{-2g+\sqrt{4g^2-4c}}{2}$$

$$x_2 = \frac{-2g-\sqrt{4g^2-4c}}{2}$$

$$x_1 = -g+\sqrt{g^2-c}$$

$$x_2 = -g-\sqrt{g^2-c}$$

length of intercept

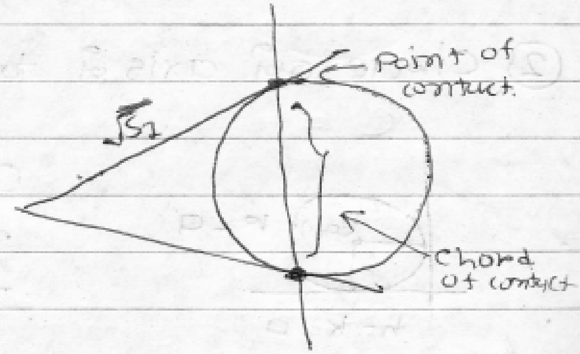
on x axis $= 2\sqrt{g^2-c}$

on y axis $= 2\sqrt{f^2-c}$

If touches x axis then $g^2=c$

If touches y axis then $f^2=c$

Pair of tangent



$S=0$

$$\boxed{S \cdot S_1 = T^2}$$

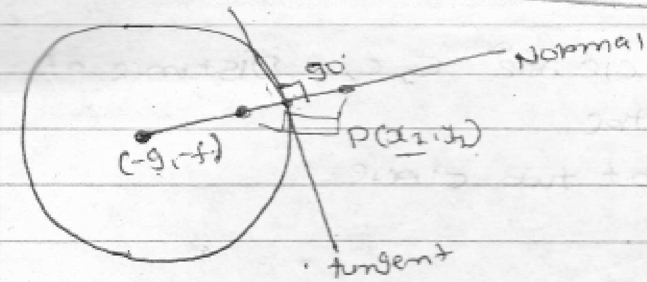
Pair of tangent

જો P circle ની બહારની બાજુ

length of tangent $= \sqrt{S_1}$

Chord of contact $= T=0$ } If P = outside

Normal and it's Equation



$$\left(\frac{y_1-y_2}{x_1-x_2}\right) = \frac{y_1-(-f)}{x_1-(-g)}$$

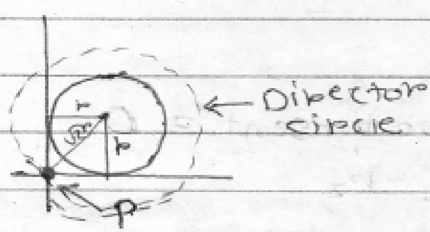
જે Normal એ Point પસાર થાય તો

$$(y-y_1) = \left(\frac{y_1+f}{x_1+g}\right)(x-x_1)$$

Director circle

⇒ Definition → "Director circle is the locus of points of intersection of two mutually perpendicular tangents of the circle"

મતલબ કે કોઈ circle ને બે tangent દોરવામાં આવે અને તે એકબીજાને લંબ હોય, તો તે એક Point પર ભેગા થાય. હવે બંને tangent ને 90° જ રાખી ને circle touch હોય તેવી રીતે જ ફેરવવામાં આવે તો એક નવું circle બને જેને -------



main circle → $x^2+y^2 = r^2$

↓

Director circle માટે

↓

$$x^2+y^2 = (\sqrt{2}\,r)^2$$

$$\boxed{x^2+y^2 = 2r^2}$$

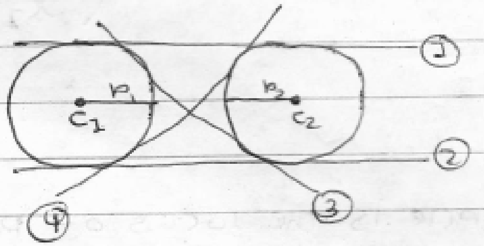
D.C. માટે Radius બદલે જ્યારે centre same જ રહે.

No. of common tangent.

let કોઈપણ બે circle માટે C_1C_2 = Distance of circle between centre

r_1 & r_2 = radius of two circle

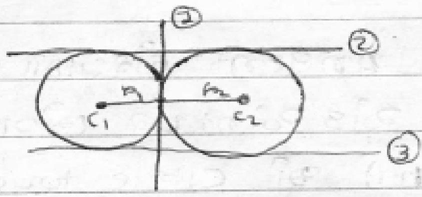
① circle is not touching



No. of common tangent $= \boxed{4}$

જ્યારે $C_1C_2 > r_1 + r_2$

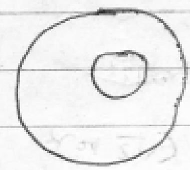
② touching externally



No. of common tangent $= \boxed{3}$

જ્યારે $C_1C_2 = r_1 + r_2$

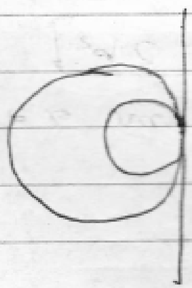
③ circle lie inside



No. of common tangent = 0

$C_1C_2 < r_1 - r_2$

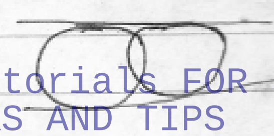
④



No. of common tangent = one

$C_1C_2 = r_1 - r_2$

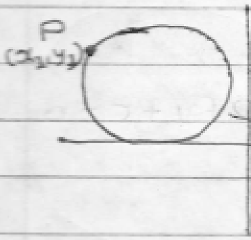
⑤



No. of common tangent = 2

$r_1 - r_2 < C_1C_2 < r_1 + r_2$

Point and circle



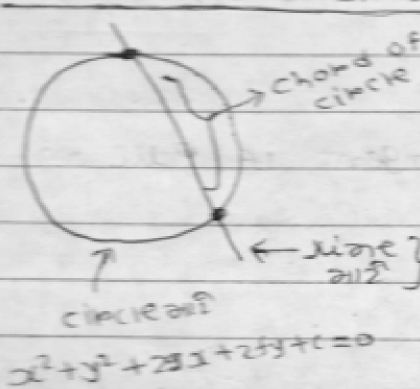
$$x^2+y^2+2gx+2fy+c=0$$
$$S=0$$
$$x_1^2+y_1^2+2gx_1+2fy_1+c$$
$$\downarrow$$
$$S_1$$

હવે જો P circle પર હોય તો $S_1 = 0$

જો P circle ની બહાર તો $S_1 > 0$

જો P circle ની અંદર તો $S_1 < 0$

Chord of a circle



અહીં એક line નું equation અને circle નું equation ભેગી solve કરતા

$y = mx + c'$

Solve કરતા $y = mx+c$ circle ના equation માં મુકતા

$$x^2 + (mx+c')^2 + 2mxc' + 2gx + 2fmx + 2fc' + c = 0$$

$$\underbrace{x^2+m^2x^2}_{x^2} + \underbrace{c'^2 + 2mxc' + 2gx + 2fmx}_{2x} + \underbrace{2fc' + c}_{c} = 0$$

It is quadratic Equation

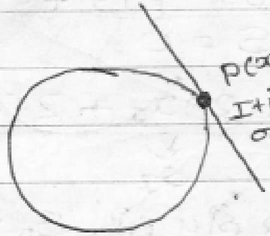
તેથી x_1, x_2 બે Root આવશે

If x_1, x_2 are real then line intersect at two Points

If $x_1 = x_2$ then line is a tangent to circle

If x_1, x_2 are imaginary then line does not cut or touch

Tangent and it's Equation



$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$S = 0$$

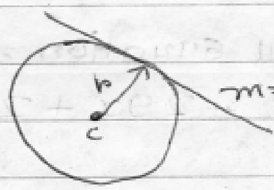
$$\left\{\begin{aligned} x^2 &= x \times x_1 \\ y^2 &= y \times y_1 \\ 2x &= x + x_1 \\ 2y &= y + y_1 \end{aligned}\right\}$$

$$x \cdot x_1 + y \cdot y_1 + g(x + x_1) + f(y + y_1) + c = 0$$

$$T = 0$$

$T = 0$ is the equation of tangent if P lie on circle

Condition of tangency



let circle નું centre = (0,0)

તો circle નું Equation

$$x^2 + y^2 = a^2$$

let line of Equation → $y = mx + c$

$$mx - y + c = 0$$

$(x, y) = (0, 0)$ મૂકતાં.

length of perpendicular p $= \dfrac{c}{\sqrt{1 + m^2}} = \pm a$

constant term

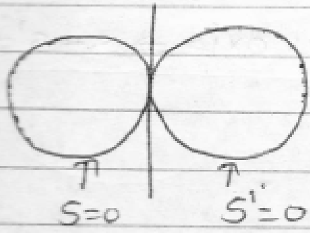
x અને y ના coefficient Root માં

Condition of tangency $\boxed{c^2 = a^2(1 + m^2)}$

a = radius

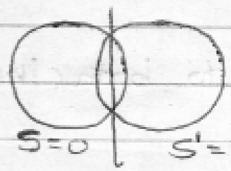
Equation of common tangent

circle touches each other.



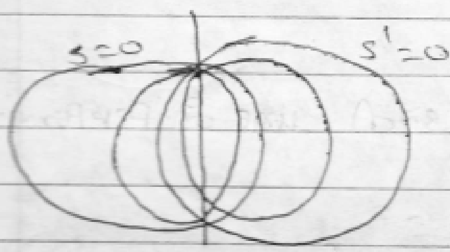
$$\boxed{S - S' = 0}$$

Common Chord કાંટપડા



$$\boxed{S - S' = 0}$$

circle through Point of intersection of two circles



~~S+S~~

$S + \lambda S' = 0$

$\boxed{\text{or}}$

$\lambda S + S' = 0$ } λ = Parameter

$x^2 + y^2 = r^2 \longrightarrow$ System of circle centre at origin or family of concentric circles

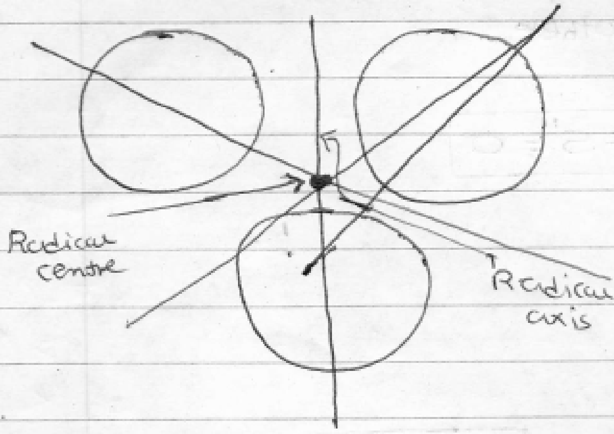
$x^2 + y^2 + 2gx + 2fy + 1 = 0$

family of circle with one Parameter

$x^2 + y^2 + 2gx + 2y + g^2 = 0$

System of circle touching x axis

Radical axis અથવા Radical centre



Radical axis $\Rightarrow S - S' = 0$

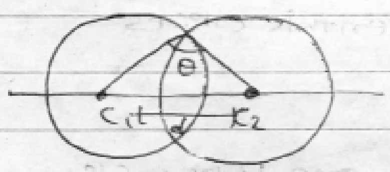
Prop. ① length of tangent to both circle from a Point is same

② It is perpendicular to line joining centre of circle

Radical axis ની centre માટે centre ની Join કરતી line ની Perpendicular હોય

Orthogonal circles

Angle between circle $= \theta$



$C_1C_2 = d$

$$\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1 r_2}$$

circle is orthogonal if $\theta = 90^\circ$

$$r_1 = \sqrt{g_1^2 + f_1^2 - c_1} \quad , \quad r_2 = \sqrt{g_2^2 + f_2^2 - c_2}$$

$$C_1C_2 = d = \sqrt{(g_1 - g_2)^2 + (f_1 - f_2)^2}$$

$$r_1^2 + r_2^2 = d^2$$

Condition of orthogonality $\longrightarrow$ $2g_1g_2 + 2f_1f_2 = c_1 + c_2$