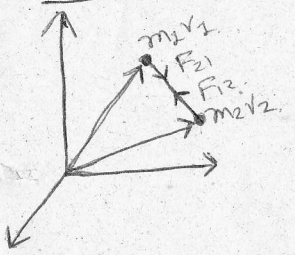
System of Particle

→ Centre of mass → It is a point where entire mass of body assumed to be concentrated.

→ also all external forces on the system of particles appear to be applied at that point.

Ex.

equation to find centre of mass



$$F = \frac{dP_1}{dt} + \frac{dP_2}{dt}$$

$$P_1 = m_1 v_1 = \frac{md}{dt} p_1$$

$$P_2 = m_2 v_2 = \frac{md}{dt} p_2$$

$$\frac{d^2}{dt^2}(m_1 p_1) + \frac{d^2}{dt^2}(m_2 p_2)$$

$$\frac{d^2 p}{dt^2} = \frac{d^2}{dt^2}\left(\frac{m_1 p_1 + m_2 p_2}{m_1 + m_2}\right)$$

$$\boxed{p_{cm} = \frac{m_1 p_1 + m_2 p_2}{m_1 + m_2}}$$

or

$$\boxed{p_{cm} = \frac{\Sigma m p}{\Sigma m}}$$

⇒ at the centre of mass any two particle's momentum is same.

⇒ The sum of moments of all masses of a system, about the centre of mass of a system is zero

→ on the body or outside the body at which the entire weight of body considered to be concentrated

$$P_{cm} = \frac{m_1 \bar{p}_1 + m_2 \bar{p}_2 + \dots + m_n \bar{p}_n}{(m_1 + m_2 + \dots + m_n)}$$

$$P_{cm} = \frac{m_1 \bar{p}_1 + m_2 \bar{p}_2 + \dots + m_n \bar{p}_n}{M}$$

$$\boxed{M P_{cm} = m_1 \bar{p}_1 + m_2 \bar{p}_2 + \dots + m_n \bar{p}_n}$$

→ hence the sum of the moments of masses about the origin is equal to the moment of the total mass of the system placed at the centre of mass.

⇒ here $p \propto \frac{1}{m} \Rightarrow \frac{p_1}{p_2} = \frac{m_2}{m_1}$

Remember

total three density.

1. linear mass density. $\lambda = M/L$
2. surface mass density $\sigma = M/A$
3. Volume mass density $\rho = M/V$

→ a quantity $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is called reduced mass of system.

Rigid body.

→ In a rigid body particles are arranged so that the distance between the particle's is fixed.

→ rigid body does not undergo changes in shape or size under action of external forces.

www.vijay-jotani.weebly.com

Centre of mass of rigid bodies

Shape	Position of centre of mass
Uniform rod →	Mid point of the rod.
circular ring →	Centre of ring
Circular disc →	Centre of disc
Cubical block →	Point of interaction of diagonals.
Cylinder →	Centre of cylinder
Solid cone →	$3/4^{th}$ of height of cone from apex on its axis
Triangular lamina →	Point of intersection of the medians

for a continuous distribution of mass the position of the centre of mass

$$\bar{R}_{cm} = \frac{\text{moments of the masses with respect to the origin}}{\text{total mass}}$$

$$= \boxed{\frac{\int \bar{r}\,dm}{m}}$$

⇒ coordinates of centre of mass ⇒ $X_{cm} = \frac{1}{m}\int x\,dm$

$Y_{cm} = \frac{1}{m}\int y\,dm$

$Z_{cm} = \frac{1}{m}\int z\,dm$

Moment of inertia and angular momentum

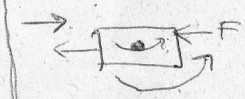
for translation motion

linear momentum $\bar{P} = m\frac{d\bar{r}}{dt}$

angular momentum $L = I\frac{d\theta}{dt} = I\omega$

$= \sum_i m_i r_i^2 \omega$

Axis of Rotation.



→ body will rotate such that all the particles of the body move along circles whose centres lie on a straight vertical line

It is called "axis of rotation"

Moment of inertia of a rigid body

→ The rotational inertia of a particle/body about an axis of rotation is called the moment of inertia of the particle/body about the axis of rotation

It depend on

1. axis of rotation
2. mass/masses of the particles which compose the system of rotating body
3. Shape of the rotating body

→ for a particle of mass m

moment of inertia $I = mr^2$

r = distance of particle from axis of rotation

for system of particle ∴ $\boxed{I = \sum_{i=1}^{i=n} m_i r_i^2}$

and for rigid body

$$\boxed{I = \int r^2\,dm}$$

www.vijay-jotani.weebly.com