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Motion under gravity

Upward motion

$S = ut - \frac{1}{2} g t^2$

$v = u - gt$

$v^2 = u^2 - 2gS$

$S_n = u - \frac{g(2n-1)}{2}$

Down ward motion

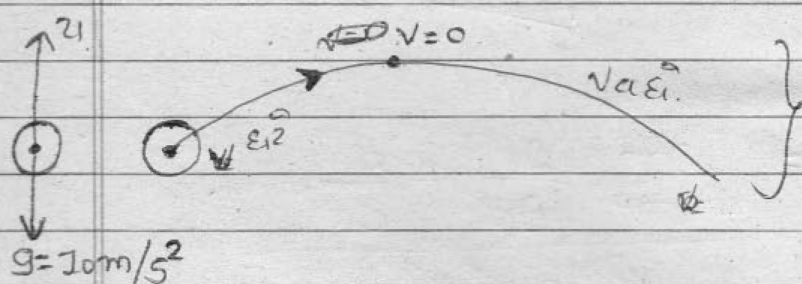
$\downarrow$ gravity

$v = u + gt$

$S = ut + \frac{1}{2} g t^2$

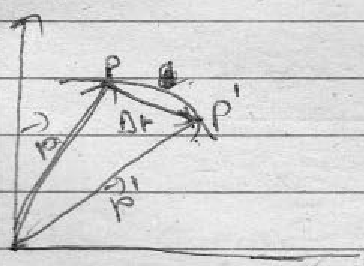
$v^2 = u^2 + 2gS$

$S_n = u + \frac{g(2n-1)}{2}$



Velocity ~~motion~~ ઝીરો
[illegible] થઈ જાય છે
maximum height = 0
[illegible]

Motion in 2D



Position vector $r = xi + yj$

After time Δt

$r' = x'i + y'j$

Displacement $= r' - r$

$\Delta r = (x'-x)i + (y'-y)j$

$V_{av} = \frac{\Delta r}{\Delta t} = \frac{\Delta x}{\Delta t} i + \frac{\Delta y}{\Delta t} j$

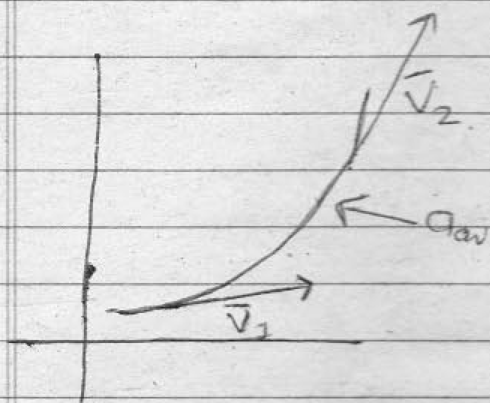
so as $t \to 0$

$\bar{V} = \frac{dx}{dt} i + \frac{dy}{dt} j = V_x i + V_y j$

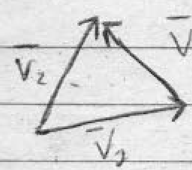
$\bar{a} = \bar{a}_x\, i + \bar{a}_y\, j$

$$\boxed{\bar{a} = \frac{d\bar{V}_x}{dt}}$$

કોઈપણ ચોક્કસ ક્ષણ ઉપરનું acceleration



$$\bar{a}_{av} = \frac{\Delta \bar{V}}{\Delta t} = \frac{\bar{V}_2 - \bar{V}_1}{\Delta t}$$



$\bar{V}_2 - \bar{V}_1 = \Delta \bar{V}$

$$a = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt}$$

Motion in Plane with constant acceleration

$$\bar{a} = \frac{\bar{V}_2 - \bar{V}_1}{\Delta t} = \frac{\Delta \bar{V}}{\Delta t}$$

$\bar{V} = \bar{u} + \bar{a} t$

$\bar{V}_x = \bar{u}_x + \bar{a}_x t$

$\bar{V}_y = \bar{u}_y + \bar{a}_y t$

$\bar{V} = \bar{V}_x\, i + \bar{V}_y\, j$

$\bar{a} = \bar{a}_x\, i + \bar{a}_y\, j$

$\bar{u} = \bar{u}_x\, i + \bar{u}_y\, j$

Note: time is common in both dimension

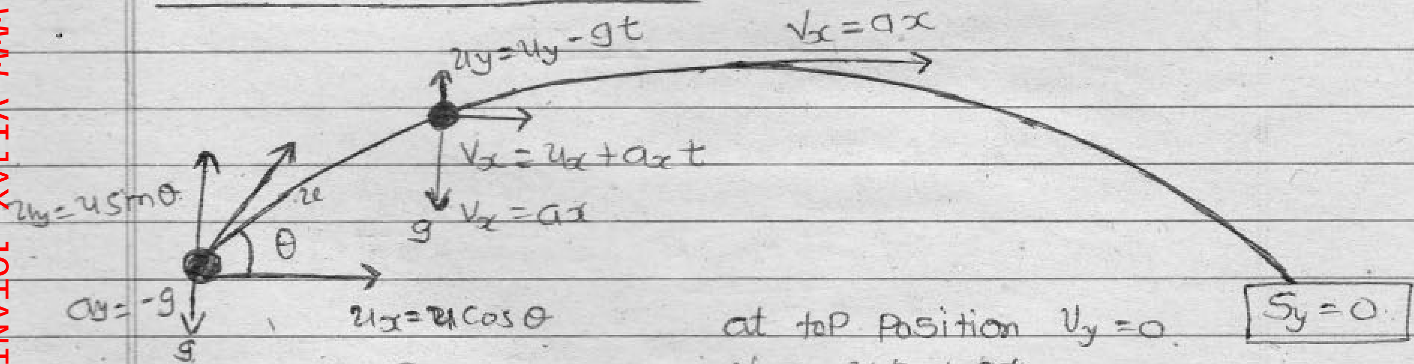
$$\bar{x} = \bar{S}_x = x_0 + \bar{u}_x t + \frac{1}{2} a_x t^2$$

$$y = \bar{S}_y = y_0 + \bar{u}_y t + \frac{1}{2} a_y t^2$$

$$\bar{v}_x^{\,2} = \bar{u}_x^{\,2} + 2 S_x a_x$$

$$\bar{v}_y^{\,2} = \bar{u}_y^{\,2} + 2 S_y a_y$$

Projectile motion



at top position $v_y = 0$

$v_y = u_y + at.$

$0 = u\sin\theta - g t_1$

$$\boxed{t_1 = \frac{u\sin\theta}{g}}$$

$$S_y = u_y t + \frac{a_y}{2} t^2$$

$$S_x = u_x t + \frac{a_x}{2} t^2.$$

Total time of flight = T

$$0 = u\sin\theta\, T - \frac{g}{2} T^2$$

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

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Range $= u_x t = (u\cos\theta)\dfrac{2u\sin\theta}{g}$

$$= \frac{u^2 2\sin\theta\cos\theta}{g} = \boxed{\frac{u^2\sin 2\theta}{g}}$$

The value of $\sin 2\theta$ is same for
$\theta = 45+\alpha$ or $45-\alpha$ & $\theta = \alpha$ or $90-\alpha$

Time taken to maximum height

$$t_m = \frac{u\sin\theta}{g}$$

Maximum height attained by Projectile

$$H = \frac{u^2\sin^2\theta}{2g}$$

$H_{max} = \dfrac{u^2}{2g}$ $\boxed{\text{when } \theta = 90^\circ}$

Time of flight $= \dfrac{\cancel{2u}\sin\theta}{\cancel{g}} = \boxed{\dfrac{2u\sin\theta}{g}}$

Range of Projectile $= \dfrac{u^2\sin 2\theta}{g}$

$$R_{max} = \frac{u^2}{g}$$

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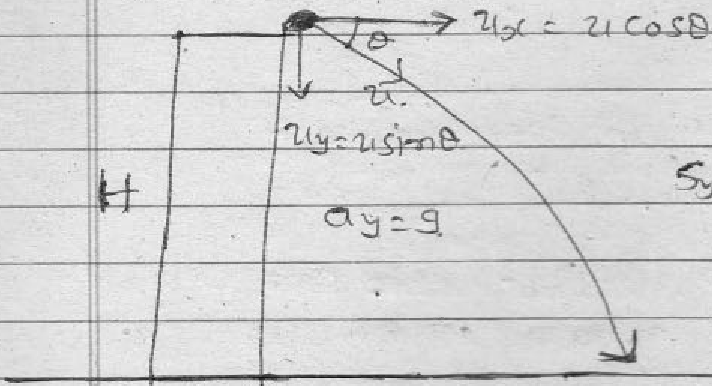
Projectiles from a height.



$$S_y = H = u\sin\theta T + \frac{1}{2} g T^2$$

$$Range = (u\cos\theta)T + 0$$

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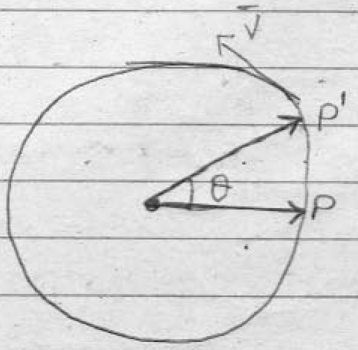
Angular variables



Angular Position = θ

unit of θ is radian

θ is Positive when anticlockwise

θ is negative when clockwise

Angular velocity = $\dfrac{d\theta}{dt} = \bar{\omega}$ (omega) → axial vector

$d\theta$ is angular Displacement

$d\theta$ is Vector if $d\theta$ is Small

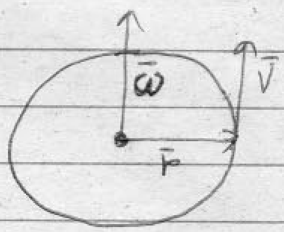
$d\theta$ is not vector if $d\theta$ is Big

$\bar{\alpha}$ = angular acceleration

$$\boxed{\alpha = \frac{d\omega}{dt}}$$

$\bar{\alpha}$ is also axial Vector

uniform circular motion



$\bar{\omega}$ = constant

$$\boxed{\bar{\omega} = 2\pi n = \frac{2\pi}{T}}$$

n = no. of terms / second

T = time interval per revolution

$\bar{r}$ = radial Vector

$\bar{\omega}$ = axial vector

$\bar{v}$ = tangential vector

$$\boxed{\bar{v} = \bar{\omega} \times \bar{r}}$$

Class Product of vector

$|v| = |\omega| |r|$ ← Scaler Product

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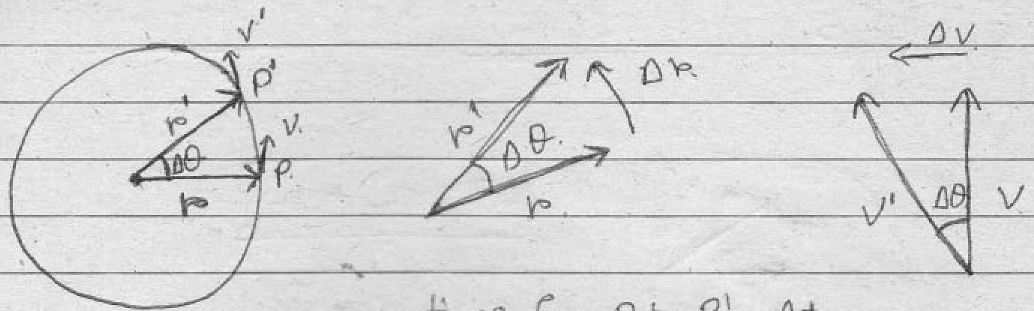
Acceleration in circular motion



time from P to P' = Δt

$$|a| = \lim_{\Delta t \to 0} \frac{|\Delta v|}{\Delta t} = \lim_{\Delta t \to 0} \frac{v}{r}\,\frac{\Delta r}{\Delta t}$$

$$\Delta\theta = \frac{\Delta r}{r} = \frac{\Delta v}{v}$$

$$= \lim_{\Delta t \to 0} \left(\frac{v}{r}\right) \frac{v\,\Delta t}{\Delta t} \qquad \left(\because\ \Delta r \simeq v\Delta t\right)$$

જો time [illegible]

$$= \boxed{\frac{v^2}{r}}$$

$$F_c = m a_c$$

$$F_c = \frac{m v^2}{r}$$

$$\boxed{a_c = \frac{v^2}{r}}$$ ← centrifugal acceleration

Acceleration

$\boxed{v = (\bar{\omega} \times \bar{r})}$ ← cross product of vectors

$$a = \frac{d\bar{v}}{dt} = \frac{\bar{\omega} \times \bar{r}}{t}$$

$$= \left(\frac{d\bar{\omega}}{dt} \times \bar{r}\right) + \left(\bar{\omega} \times \frac{d\bar{r}}{dt}\right)$$

$$= (\alpha \times \bar{r}) + \left(\bar{\omega} \times \frac{d\theta}{dt} \times r\right)$$

$$= (\alpha \times \bar{r}) + (\bar{\omega} \times \ldots\ \bar{r} \times \bar{\omega} - (\bar{\omega}\cdot\bar{\omega})\bar{r}$$

$$= (\alpha + \bar{r}) + \bar{\omega} \times (\bar{\omega} \times \bar{r})$$

$$= (\alpha + \bar{r}) + (\ldots)$$

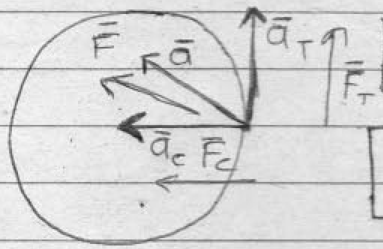
$$= (\alpha + \bar{r}) + (-\bar{\omega}^2 \cdot r)$$

$$a = a_T + a_c$$

$$\boxed{a_c = \ldots^2}$$

FORCES IN CIRCULAR MOTION

$\bar{F} = m\bar{a}$



$\boxed{\bar{a} = a_T + a_c}$

$\boxed{a_T = \alpha \times \bar{r}}$

$a_T = \alpha r$

speed $v = \omega r$

$a_T = \frac{d\omega}{dt} r$

Tengential acceleration is due to change in speed of particle.

centripital acceleration is due to change in direction motion of particle

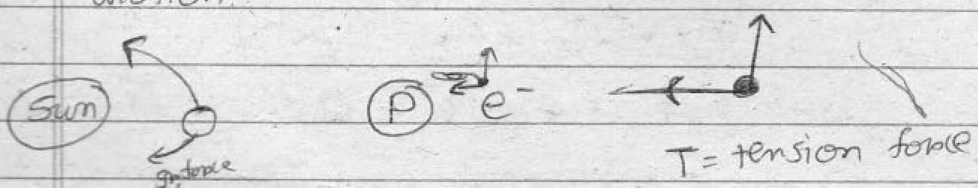
$\boxed{a_T = \frac{dv}{dt} = \frac{d\omega}{dt} r = \alpha r}$

$\boxed{a_c = \frac{v^2}{r} = \omega^2 r = v\omega}$ કોઈપણ formula થી centripital acceleration find કરી શકાય.

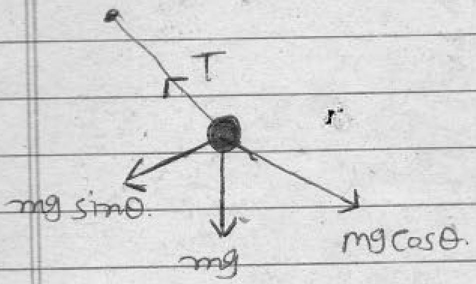
CENTRIPITAL FORCE

uniform circular motion. ω = constant

Centripital force is required for a circular motion



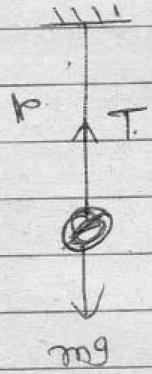
T = tension force



$T - mg\cos\theta =$ resultant force towards centre

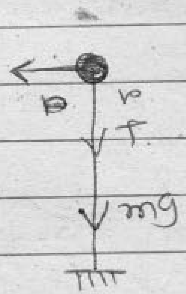
$= \text{centripital force} = \dfrac{mv^2}{r}$

$= m\omega^2 r = mv\omega$



Resultant force towards centre $= T - mg$

$= F_c = \dfrac{mv^2}{r}$



Resultant force towards centre $=$

$= T + mg = \dfrac{mv^2}{r}$

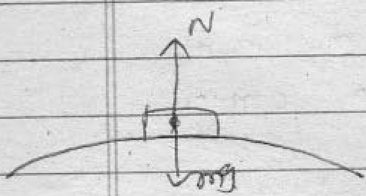
For a complete circle T must be

$T \geq 0$

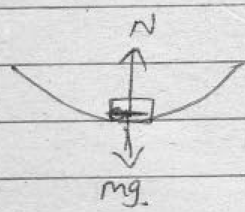
So minimum V at top.

$mg = \dfrac{mv_{min}^2}{r}$

$V_{min} = \sqrt{gr}$



$mg - N = \dfrac{mv^2}{r}$



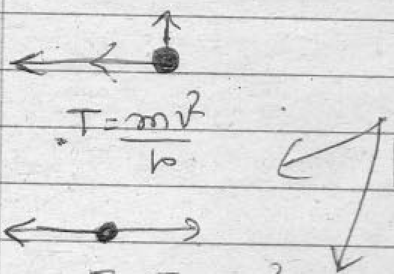
$N - mg = \dfrac{mv^2}{r}$

CENTRIFUGAL FORCE

centrifugal force is a Pseudo force

Pseudo force $= -ma$

centrifugal force $= \dfrac{mv^2}{r}$



$T = \dfrac{mv^2}{r}$

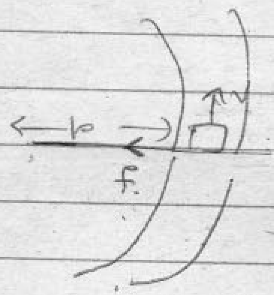
$\boxed{\dfrac{mv^2}{r} = F}$

both are same

$F = T = \dfrac{mv^2}{r}$

force નું Equation છે. rotating frame of refference માં છે. ground frame of reference માં લખીએ તો same થાય.

CIRCULAR TURNING ON ROAD

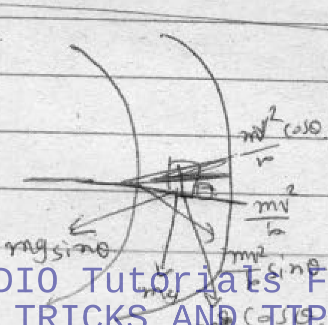


$$F_c = \frac{mv^2}{r} \le \mu mg$$

f = force of friction = maximum (μmg)

$$\frac{mv^2_{max}}{r} = \mu mg$$

$$V_{max} = \sqrt{\mu g r}$$



$$mg\sin\theta = \frac{mv^2}{r}\cos\theta$$

$$\boxed{\tan\theta = v^2/gr}$$

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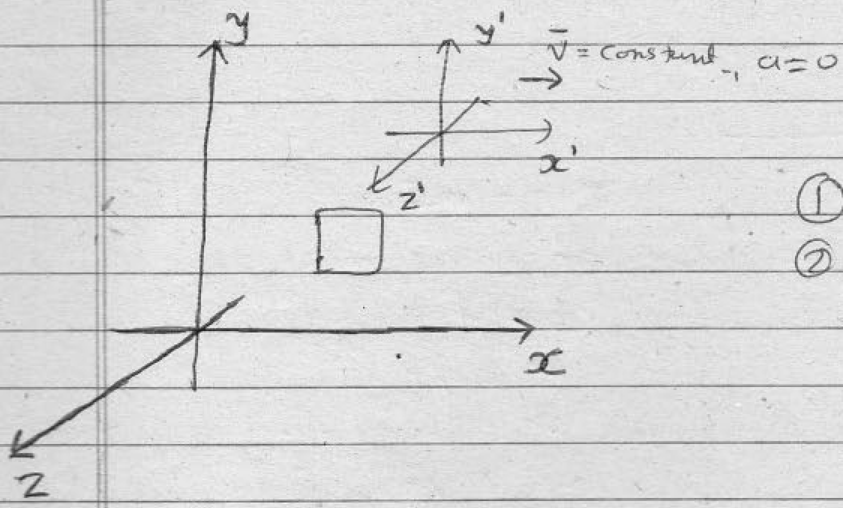
FRAME OF REFERENCE



① Inertial frame of ref.
② Non Inertial frame of ref.

Inertial frame of ref.

A ref. frame is an inertial frame of ref. if it moves with zero acceleration w.r.t. other inertial frame of reference

→ (A frame is inertial if Newton's law is applicable in it)

Our earth is a rotating ref. frame. So it a non inertial frame of ref.

Non Inertial frame of ref.

A ref. frame is an non inertial frame of ref. if it moves with some acceleration w.r.t. an inertial frame of ref.

(Newton's is not applicable in non inertial frame of ref)

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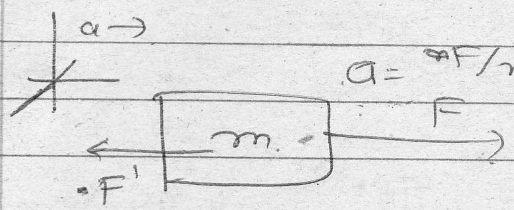
Pseudo force



$a = F/m$

$F' = -m\bar{a}$

where, m is the mass of object under consederation

a = Acceleration of Non Inertial R.f

a = 0

In ref. frame of mas

m'

$(F - F') = 0$

same

$F = ma$

$F - ma = 0$